

We also see that when:  $\Delta\varphi_{BA} = \pm 1\pi, \pm 3\pi, \pm 5\pi, \dots = \pm n_{\text{odd}}\pi$

since:  $\cos(\theta \pm n_{\text{odd}}\pi) = (\cos\theta \cdot \cos n_{\text{odd}}\pi) \mp (\sin\theta \cdot \sin n_{\text{odd}}\pi) = -\cos\theta$ , the **total** energy density is **all** in the form of **kinetic** energy density:

$$w_{\text{pot}}^{\text{inst}}(x=0, t) = \frac{1}{2} \frac{|\tilde{A}|^2}{\rho_o c^2} [\cos \omega t + \cos(\omega t \pm n_{\text{odd}}\pi)]^2 = 0$$

$$w_{\text{kin}}^{\text{inst}}(x=0, t) = \frac{1}{2} \frac{|\tilde{A}|^2}{\rho_o c^2} [\cos \omega t - \cos(\omega t + \Delta\varphi_{BA}^o)]^2 = \frac{1}{2} \frac{4|\tilde{A}|^2}{\rho_o c^2} \cos^2 \omega t = \frac{2|\tilde{A}|^2}{\rho_o c^2} \cos^2 \omega t$$

$$w_{\text{tot}}^{\text{inst}}(x=0, t) \equiv w_{\text{pot}}^{\text{inst}}(x=0, t) + w_{\text{kin}}^{\text{inst}}(x=0, t) = \frac{2|\tilde{A}|^2}{\rho_o c^2} \cos^2 \omega t = \frac{2|\tilde{A}|^2}{z_o c} \cos^2 \omega t$$

The **time-averaged** potential, kinetic and total energy densities associated with two counter-propagating 1-D monochromatic traveling plane waves are:

$$\langle w_{\text{pot}}(x, t) \rangle_t = \frac{1}{2} \frac{\langle p_{\text{tot}}^2(x, t) \rangle_t}{\rho_o c^2} = \frac{1}{4} \frac{|\tilde{A}|^2}{\rho_o c^2} [1 + |\tilde{R}|^2 + 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o)]$$

$$\langle w_{\text{kin}}(x, t) \rangle_t = \frac{1}{2} \rho_o \langle u_{\text{tot}}^{\parallel 2}(x, t) \rangle_t = \frac{1}{4} \frac{|\tilde{A}|^2}{\rho_o c^2} [1 + |\tilde{R}|^2 - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o)]$$

$$\langle w_{\text{tot}}(x, t) \rangle_t \equiv \langle w_{\text{pot}}(x, t) \rangle_t + \langle w_{\text{kin}}(x, t) \rangle_t = \frac{1}{2} \frac{|\tilde{A}|^2}{\rho_o c^2} [1 + |\tilde{R}|^2] = \frac{1}{2} \frac{|\tilde{A}|^2}{z_o c} [1 + |\tilde{R}|^2]$$

Note **here**, that the ratio of the **time-averaged potential** energy density to the **time-averaged kinetic** energy density is **not** equal to unity for counter-propagating monochromatic plane waves:

$$\frac{\langle w_{\text{pot}}(x, t) \rangle_t}{\langle w_{\text{kin}}(x, t) \rangle_t} = \frac{\frac{1}{2} \frac{\langle p_{\text{tot}}^2(x, t) \rangle_t}{\rho_o c^2}}{\frac{1}{2} \rho_o \langle u_{\text{tot}}^{\parallel 2}(x, t) \rangle_t} = \frac{[1 + |\tilde{R}|^2 + 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o)]}{[1 + |\tilde{R}|^2 - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o)]} \neq 1$$

Again, for an observer's position at  $x = 0$  **and**.  $|\tilde{R}| \equiv |\tilde{B}|/|\tilde{A}| = 1$  (i.e. a "pure" standing wave), these quantities reduce to:

$$\langle w_{\text{pot}}(x=0, t) \rangle_t = \frac{1}{2} \frac{\langle p_{\text{tot}}^2(x=0, t) \rangle_t}{\rho_o c^2} = \frac{1}{2} \frac{|\tilde{A}|^2}{\rho_o c^2} [1 + \cos \Delta\varphi_{BA}^o]$$

$$\langle w_{\text{kin}}(x=0, t) \rangle_t = \frac{1}{2} \rho_o \langle u_{\text{tot}}^{\parallel 2}(x=0, t) \rangle_t = \frac{1}{2} \frac{|\tilde{A}|^2}{\rho_o c^2} [1 - \cos \Delta\varphi_{BA}^o]$$