

and we see that:

$$\begin{aligned}\varphi_{z_{a_{tot}}}(x, \omega) &\equiv \tan^{-1} \left( \frac{\text{Im} \left\{ \tilde{z}_{a_{tot}}^{\parallel}(x, \omega) \right\}}{\text{Re} \left\{ \tilde{z}_{a_{tot}}^{\parallel}(x, \omega) \right\}} \right) = \tan^{-1} \left( \frac{2|\tilde{R}|\sin(2kx + \Delta\varphi_{BA}^o)}{\{1 - |\tilde{R}|^2\}} \right) \\ &= \Delta\varphi_{p_{tot} - u_{tot}^{\parallel}}(x, \omega) = \varphi_{p_{tot}}(x, \omega) - \varphi_{u_{tot}^{\parallel}}(x, \omega) = \\ \varphi_{c_{a_{tot}}}(x, \omega) &\equiv \tan^{-1} \left( \frac{\text{Im} \left\{ \tilde{c}_{a_{tot}}^{\parallel}(x, \omega) \right\}}{\text{Re} \left\{ \tilde{c}_{a_{tot}}^{\parallel}(x, \omega) \right\}} \right)\end{aligned}$$

Hence, we also see that:

$$\begin{aligned}\varphi_{I_{a_{tot}}}(x, \omega) &= \varphi_{z_{a_{tot}}}(x, \omega) = \varphi_{c_{a_{tot}}}(x, \omega) = \Delta\varphi_{p_{tot} - u_{tot}^{\parallel}}(x, \omega) = \varphi_{p_{tot}}(x, \omega) - \varphi_{u_{tot}^{\parallel}}(x, \omega) \\ &= \tan^{-1} \left( \frac{2|\tilde{R}|\sin(2kx + \Delta\varphi_{BA}^o)}{\{1 - |\tilde{R}|^2\}} \right)\end{aligned}$$

When  $|\tilde{R}| \equiv |\tilde{B}|/|\tilde{A}| = 1$  (i.e. a **pure** standing wave!), then:

$$\tilde{I}_{a_{tot}}^{\parallel}(x, \omega) \equiv \frac{1}{2} \tilde{p}_{tot}(x, \omega) \cdot \tilde{u}_{tot}^{\parallel*}(x, \omega) = \frac{1}{2} \frac{|\tilde{A}|^2}{z_o} \left[ \{1 - |\tilde{R}|^2\} + 2i|\tilde{R}|\sin(2kx + \Delta\varphi_{BA}^o) \right] = \tilde{I}_{a_{tot}r}^{\parallel}(x, \omega) + i\tilde{I}_{a_{tot}i}^{\parallel}(x, \omega)$$

For an observer/listener's position at  $x = 0$  **and**  $|\tilde{R}| = 1$ , this reduces to:

$$\tilde{I}_{a_{tot}}^{\parallel}(x = 0, \omega) \equiv \frac{1}{2} \tilde{p}_{tot}(x = 0, \omega) \cdot \tilde{u}_{tot}^{\parallel*}(x = 0, \omega) = i \frac{|\tilde{A}|^2}{z_o} \sin \Delta\varphi_{BA}^o \text{ (n.b. **purely imaginary quantity!**)}$$

We see again that when additionally:  $\Delta\varphi_{BA}^o = 0, \pm\pi, \pm2\pi, \pm3\pi, \dots = \pm n\pi$  that:  $\tilde{I}_{a_{tot}}^{\parallel}(x = 0, t) = 0$  !!!

Similarly, we see that  $\tilde{I}_{a_{tot}}^{\parallel}(x = 0, \omega)$  has a purely **imaginary extremum** amplitude of

$$\pm |\tilde{A}|^2 / \rho_o c = \pm |\tilde{A}|^2 / z_o \text{ when } \Delta\varphi_{AB} = \pm\pi/2, \pm3\pi/2, \pm5\pi/2, \dots = \pm m_{\text{odd}}\pi/2.$$

Physically, the **real** part of the complex **frequency-domain** longitudinal acoustic intensity  $\tilde{I}_{a_{tot}}^{\parallel}(x, \omega)$  represents the **frequency-domain** “amplitude” of the **net flux/flow** of acoustic energy crossing unit area per unit time (SI units *Watts/m<sup>2</sup>*) – i.e. the **real** part of the complex acoustic intensity is physically associated with **propagating** sound/sound **radiation**. The **imaginary** part of the complex **frequency-domain** longitudinal acoustic intensity is physically associated with **non-propagating** acoustic energy, i.e. energy sloshing back and forth each cycle of oscillation.