

The **phase** associated with the complex **frequency-domain**  $\tilde{I}_{a_{tot}}^{\parallel}(x, \omega)$  is:

$$\varphi_{I_{a_{tot}}}(x, \omega) \equiv \tan^{-1} \left( \frac{\text{Im} \left\{ \tilde{I}_{a_{tot}}^{\parallel}(x, \omega) \right\}}{\text{Re} \left\{ \tilde{I}_{a_{tot}}^{\parallel}(x, \omega) \right\}} \right) = \tan^{-1} \left( \frac{\frac{1}{2} \frac{|\tilde{A}|^2}{z_o} \left[ 2|\tilde{R}| \sin(2kx + \Delta\varphi_{BA}^o) \right]}{\frac{1}{2} \frac{|\tilde{A}|^2}{z_o} \left[ \{1 - |\tilde{R}|^2\} \right]} \right) = \tan^{-1} \left( \frac{\left[ 2|\tilde{R}| \sin(2kx + \Delta\varphi_{BA}^o) \right]}{\left[ \{1 - |\tilde{R}|^2\} \right]} \right)$$

Compare the above **frequency-domain** total/resultant complex longitudinal acoustic intensity expressions to those associated with the complex longitudinal **specific** acoustic impedance and complex longitudinal energy flow velocity:

$$\frac{\tilde{z}_{a_{tot}}^{\parallel}(x, \omega)}{z_o} = \frac{\left[ \{1 - |\tilde{R}|^2\} + 2i|\tilde{R}| \sin(2kx + \Delta\varphi_{BA}^o) \right]}{\left[ \{1 + |\tilde{R}|^2\} - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o) \right]} = \frac{\tilde{c}_{a_{tot}}^{\parallel}(x, \omega)}{c}$$

Since:  $\tilde{I}_a \equiv \frac{1}{2} \tilde{p} \tilde{u}^*$  and  $\tilde{z}_a \equiv \frac{\tilde{p}}{\tilde{u}} = \frac{\tilde{p} \cdot \tilde{u}^*}{\tilde{u} \cdot \tilde{u}^*} = \frac{\tilde{p} \tilde{u}^*}{|\tilde{u}|^2} = \frac{2\tilde{I}_a}{|\tilde{u}|^2}$ , or:  $\tilde{I}_a(x, \omega) = \frac{1}{2} |\tilde{u}(x, \omega)|^2 \tilde{z}_a(x, \omega)$ ,

For the situation **here** with counter-propagating 1-D monochromatic traveling plane waves, and using  $k = \omega/c$ :

$$|\tilde{u}_{tot}^{\parallel}(x, \omega)|^2 = \frac{|\tilde{A}|^2}{z_o^2} \left[ 1 - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}) + |\tilde{R}|^2 \right] = \frac{|\tilde{A}|^2}{z_o^2} \left[ \{1 + |\tilde{R}|^2\} - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}) \right]$$

Thus we see that, indeed:

$$\frac{2\tilde{I}_{a_{tot}}^{\parallel}(x, \omega)}{|\tilde{u}_{tot}^{\parallel}(x, \omega)|^2} = \frac{\frac{1}{2} \frac{|\tilde{A}|^2}{z_o} \left[ \{1 - |\tilde{R}|^2\} + 2i|\tilde{R}| \sin(2kx + \Delta\varphi_{BA}^o) \right]}{\frac{|\tilde{A}|^2}{z_o^2} \left[ \{1 + |\tilde{R}|^2\} - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o) \right]} = z_o \frac{\left[ \{1 - |\tilde{R}|^2\} + 2i|\tilde{R}| \sin(2kx + \Delta\varphi_{BA}^o) \right]}{\left[ \{1 + |\tilde{R}|^2\} - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o) \right]} = \tilde{z}_{a_{tot}}^{\parallel}(x, \omega)$$

i.e. that:

$$\frac{2\tilde{I}_{a_{tot}}^{\parallel}(x, \omega)}{|\tilde{u}_{tot}^{\parallel}(x, \omega)|^2 z_o} = \frac{\left[ \{1 - |\tilde{R}|^2\} + 2i|\tilde{R}| \sin(2kx + \Delta\varphi_{BA}^o) \right]}{\left[ \{1 + |\tilde{R}|^2\} - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o) \right]} = \frac{\tilde{z}_{a_{tot}}^{\parallel}(x, \omega)}{z_o} = \frac{\tilde{c}_{a_{tot}}^{\parallel}(x, \omega)}{c}$$