

$$\begin{aligned} \frac{|\tilde{z}_{a\text{ tot}}^{\parallel}(x)|_{\text{minima}}}{z_o} &= \frac{\sqrt{\{1-|\tilde{R}|^2\}^2 + 4|\tilde{R}|^2 \sin^2(2kx + \Delta\varphi_{BA}^o)}}{\left[\{1+|\tilde{R}|^2\} - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o) \right]} = \frac{\sqrt{\{1-|\tilde{R}|^2\}^2}}{\left[\{1+|\tilde{R}|^2\} + 2|\tilde{R}| \right]} \\ &= \left(\frac{1-|\tilde{R}|^2}{1+2|\tilde{R}|+|\tilde{R}|^2} \right) = \left(\frac{1-|\tilde{R}|^2}{(1+|\tilde{R}|)^2} \right) = \frac{(1-|\tilde{R}|) \cdot \cancel{(1+|\tilde{R}|)}}{(1+|\tilde{R}|) \cdot \cancel{(1+|\tilde{R}|)}} = \frac{(1-|\tilde{R}|)}{(1+|\tilde{R}|)} = \frac{|\tilde{c}_{a\text{ tot}}^{\parallel}(x)|_{\text{minima}}}{c} \end{aligned}$$

The phase(s) associated with the complex longitudinal **specific** acoustic impedance and complex energy flow velocity **minima** are:

$$\begin{aligned} \varphi_{z_a}(x)|_{\text{minima}} &= \tan^{-1} \left(\frac{2|\tilde{R}| \sin(2kx + \Delta\varphi_{BA}^o)}{\{1-|\tilde{R}|^2\}} \right) = \tan^{-1}(0) = 0 \\ &= \Delta\varphi_{p_{\text{tot}}-u_{\text{tot}}}(x)|_{\text{minima}} = (\varphi_{p_{\text{tot}}}(x) - \varphi_{u_{\text{tot}}}(x))|_{\text{minima}} = \varphi_{c_a}(x)|_{\text{minima}} \end{aligned}$$

Thus, for longitudinal **specific** acoustic impedance and longitudinal energy flow velocity **minima** associated with this situation, we see that the total/resultant complex pressure $\tilde{p}_{\text{tot}}(x,t)$ and longitudinal particle velocity $\tilde{u}_{\text{tot}}^{\parallel}(x,t)$ are precisely **out-of-phase** with each other, or at least by \pm **odd** integer multiples of π . Since $|\tilde{z}_{\text{tot}}^{\parallel}(x)|_{\text{minima}} = |\tilde{p}_{\text{tot}}(x,t)|/|\tilde{u}_{\text{tot}}^{\parallel}(x,t)|_{\text{minima}}$ this also tells us that whenever $(2kx + \Delta\varphi_{BA}) = \pm 1\pi, \pm 3\pi, \pm 5\pi \dots = \pm n_{\text{odd}}\pi$, the magnitude of the total/resultant complex pressure $|\tilde{p}_{\text{tot}}(x,t)|$ will also be a **minima**, whereas the magnitude of the total/resultant complex longitudinal particle velocity $|\tilde{u}_{\text{tot}}^{\parallel}(x,t)|$ will simultaneously be a **maxima**:

$$\begin{aligned} \tilde{p}_{\text{tot}}(x,t) &= \tilde{A} \left[1 + |\tilde{R}| e^{i(2kx + \Delta\varphi_{BA}^o)} \right] \cdot e^{i(\omega t - kx)} = \tilde{A} \left[1 + |\tilde{R}| e^{\pm i n_{\text{odd}}\pi} \right] \cdot e^{i(\omega t - kx)} \\ &= \tilde{A} \left[1 + |\tilde{R}| \left\{ \cos(\pm n_{\text{odd}}\pi) + i \cancel{\sin(\pm n_{\text{odd}}\pi)} \right\} \right] \cdot e^{i(\omega t - kx)} = \tilde{A} \left[1 - |\tilde{R}| \right] \cdot e^{i(\omega t - kx)} \end{aligned}$$

and:

$$\begin{aligned} \tilde{u}_{\text{tot}}^{\parallel}(x,t) &= \frac{\tilde{A}}{\rho_o c} \left[1 - |\tilde{R}| e^{i(2kx + \Delta\varphi_{BA}^o)} \right] \cdot e^{i(\omega t - kx)} = \frac{\tilde{A}}{\rho_o c} \left[1 - |\tilde{R}| e^{\pm i n_{\text{odd}}\pi} \right] \cdot e^{i(\omega t - kx)} \\ &= \frac{\tilde{A}}{\rho_o c} \left[1 - |\tilde{R}| \left\{ \cos(\pm n_{\text{odd}}\pi) + i \cancel{\sin(\pm n_{\text{odd}}\pi)} \right\} \right] \cdot e^{i(\omega t - kx)} = \frac{\tilde{A}}{\rho_o c} \left[1 + |\tilde{R}| \right] \cdot e^{i(\omega t - kx)} \end{aligned}$$

$$\begin{aligned} \Rightarrow |\tilde{p}_{\text{tot}}(x,t)| &\equiv \sqrt{\tilde{p}_{\text{tot}}(x,t) \cdot \tilde{p}_{\text{tot}}^*(x,t)} = |\tilde{A}| \left[1 - |\tilde{R}| \right] & \left\{ \begin{array}{l} \text{for } |\tilde{R}| = 1: |\tilde{p}_{\text{tot}}(x,t)| = 0 \\ \text{for } |\tilde{R}| = 1: |\tilde{u}_{\text{tot}}^{\parallel}(x,t)| = \frac{2|\tilde{A}|}{\rho_o c} \end{array} \right. & \boxed{\text{“Pure” standing wave!!!}} \\ \Rightarrow |\tilde{u}_{\text{tot}}^{\parallel}(x,t)| &\equiv \sqrt{\tilde{u}_{\text{tot}}^{\parallel}(x,t) \cdot \tilde{u}_{\text{tot}}^{\parallel*}(x,t)} = \frac{|\tilde{A}|}{\rho_o c} \left[1 + |\tilde{R}| \right] \end{aligned}$$