

The phase(s) associated with the complex longitudinal ***specific*** acoustic impedance and complex longitudinal energy flow velocity ***maxima*** occur when:

$$\begin{aligned}\varphi_{z_a}(x)\Big|_{\text{maxima}} &= \tan^{-1} \left(\frac{2|\tilde{R}|\sin(2kx + \Delta\varphi_{BA}^o)}{\{1 - |\tilde{R}|^2\}} \right) = \tan^{-1}(0) = 0 \\ &= \Delta\varphi_{p_{tot} - u_{tot}^{\parallel}}(x)\Big|_{\text{maxima}} = \left(\varphi_{p_{tot}}(x) - \varphi_{u_{tot}^{\parallel}}(x) \right)\Big|_{\text{maxima}} = \varphi_{c_a}(x)\Big|_{\text{maxima}}\end{aligned}$$

Thus, for longitudinal ***specific*** acoustic impedance and longitudinal energy flow velocity ***maxima*** associated with this situation, we see that the total/resultant complex pressure $\tilde{p}_{tot}(x, t)$ and longitudinal particle velocity $\tilde{u}_{tot}^{\parallel}(x, t)$ are precisely ***in-phase*** with each other, or at least by \pm ***even*** integer multiples of π .

Since $\left| \tilde{z}_{tot}^{\parallel}(x) \right|_{\text{maxima}} = \left| \tilde{p}_{tot}(x, t) \right| / \left| \tilde{u}_{tot}^{\parallel}(x, t) \right|_{\text{maxima}}$ this also tells us that whenever $(2kx + \Delta\varphi_{BA}^o) = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi \dots = \pm n_{\text{even}}\pi$, the ***magnitude*** of the total/resultant complex pressure $\left| \tilde{p}_{tot}(x, t) \right|$ will also be a ***maxima***, whereas the ***magnitude*** of the total/resultant complex longitudinal particle velocity $\left| \tilde{u}_{tot}^{\parallel}(x, t) \right|$ will simultaneously be a ***minima***:

$$\begin{aligned}\tilde{p}_{tot}(x, t) &= \tilde{A} \left[1 + |\tilde{R}| e^{i(2kx + \Delta\varphi_{BA}^o)} \right] \cdot e^{i(\omega t - kx)} = \tilde{A} \left[1 + |\tilde{R}| e^{\pm i n_{\text{even}}\pi} \right] \cdot e^{i(\omega t - kx)} \\ &= \tilde{A} \left[1 + |\tilde{R}| \left\{ \cos(\pm n_{\text{even}}\pi) + i \sin(\pm n_{\text{even}}\pi) \right\} \right] \cdot e^{i(\omega t - kx)} = \tilde{A} \left[1 + |\tilde{R}| \right] \cdot e^{i(\omega t - kx)}\end{aligned}$$

and:

$$\begin{aligned}\tilde{u}_{tot}^{\parallel}(x, t) &= \frac{\tilde{A}}{\rho_o c} \left[1 - |\tilde{R}| e^{i(2kx + \Delta\varphi_{BA}^o)} \right] \cdot e^{i(\omega t - kx)} = \frac{\tilde{A}}{\rho_o c} \left[1 - |\tilde{R}| e^{\pm i n_{\text{even}}\pi} \right] \cdot e^{i(\omega t - kx)} \\ &= \frac{\tilde{A}}{\rho_o c} \left[1 - |\tilde{R}| \left\{ \cos(\pm n_{\text{even}}\pi) + i \sin(\pm n_{\text{even}}\pi) \right\} \right] \cdot e^{i(\omega t - kx)} = \frac{\tilde{A}}{\rho_o c} \left[1 - |\tilde{R}| \right] \cdot e^{i(\omega t - kx)}\end{aligned}$$

$$\begin{aligned}\Rightarrow \left| \tilde{p}_{tot}(x, t) \right| &\equiv \sqrt{\tilde{p}_{tot}(x, t) \cdot \tilde{p}_{tot}^*(x, t)} = \tilde{A} \left[1 + |\tilde{R}| \right] & \left\{ \begin{array}{l} \text{for } |\tilde{R}| = 1: \left| \tilde{p}_{tot}(x, t) \right| = 2|\tilde{A}| \\ \text{for } |\tilde{R}| = 1: \left| \tilde{u}_{tot}^{\parallel}(x, t) \right| = 0 \end{array} \right. & \boxed{\text{“Pure” standing wave!!!}} \\ \Rightarrow \left| \tilde{u}_{tot}^{\parallel}(x, t) \right| &\equiv \sqrt{\tilde{u}_{tot}^{\parallel}(x, t) \cdot \tilde{u}_{tot}^{\parallel*}(x, t)} = \frac{\tilde{A}}{\rho_o c} \left[1 - |\tilde{R}| \right]\end{aligned}$$

In general, for arbitrary values of x , ***minima*** of the complex longitudinal ***specific*** acoustic impedance $\tilde{z}_{a_{tot}}^{\parallel}(x)$ and the complex longitudinal energy flow velocity $\tilde{c}_{a_{tot}}^{\parallel}(x)$ will occur whenever $(2kx + \Delta\varphi_{BA}^o) = \pm 1\pi, \pm 3\pi, \pm 5\pi \dots = \pm n_{\text{odd}}\pi$, *i.e.* whenever $\cos(2kx + \Delta\varphi_{BA}^o) = -1$, and $\sin(2kx + \Delta\varphi_{BA}^o) = 0$, then: