

The **phase** of the complex longitudinal **specific** acoustic impedance and longitudinal energy flow velocity associated with the two counter-propagating 1-D monochromatic plane waves is:

$$\begin{aligned} \varphi_{z_{a_{tot}}}(x) &\equiv \tan^{-1} \left(\frac{\text{Im} \left\{ \tilde{z}_{a_{tot}}^{\parallel}(x) \right\}}{\text{Re} \left\{ \tilde{z}_{a_{tot}}^{\parallel}(x) \right\}} \right) = \tan^{-1} \left(\frac{\cancel{z_o} \left[\frac{2|\tilde{R}| \sin(2kx + \Delta\varphi_{BA}^o)}{\left[\left\{ 1 + |\tilde{R}|^2 \right\} - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o) \right]} \right]}{\cancel{z_o} \left[\frac{\left\{ 1 - |\tilde{R}|^2 \right\}}{\left[\left\{ 1 + |\tilde{R}|^2 \right\} - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o) \right]} \right]} \right) \\ &= \tan^{-1} \left(\frac{2|\tilde{R}| \sin(2kx + \Delta\varphi_{BA}^o)}{\left\{ 1 - |\tilde{R}|^2 \right\}} \right) = \Delta\varphi_{p_{tot} - u_{tot}}^{\parallel}(x) = \varphi_{p_{tot}}(x) - \varphi_{u_{tot}}(x) = \varphi_{c_{a_{tot}}}(x) \end{aligned}$$

Thus, *e.g.* for an observer/listener's position $x = 0$, **.and.** for **equal-strength** pressure amplitudes $|\tilde{A}| = |\tilde{B}| \Rightarrow |\tilde{R}| \equiv |\tilde{B}|/|\tilde{A}| = 1$ (*i.e.* a "**pure**" standing wave) these two formulae simplify to:

$$\tilde{z}_{a_{tot}}^{\parallel}(x=0) = z_o \frac{i \sin \Delta\varphi_{BA}^o}{\left[1 - \cos \Delta\varphi_{BA}^o \right]}, \text{ or: } \tilde{c}_{a_{tot}}^{\parallel}(x=0) = c \frac{i \sin \Delta\varphi_{BA}^o}{\left[1 - \cos \Delta\varphi_{BA}^o \right]}$$

and:

$$\begin{aligned} \varphi_{z_a}(x=0) = \varphi_{c_a}(x=0) &= \tan^{-1} \left(\frac{2 \sin \Delta\varphi_{BA}^o}{0} \right) = \tan^{-1}(\pm\infty) \\ &= \Delta\varphi_{p_{tot} - u_{tot}}(x=0) = \varphi_{p_{tot}}(x=0) - \varphi_{u_{tot}}(x=0) \\ &= \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2, \dots = \pm n_{\text{odd}} \pi/2 \end{aligned}$$

i.e. for an observer/listener's position $x = 0$, **.and.** for **equal-strength** pressure amplitudes $|\tilde{A}| = |\tilde{B}| \Rightarrow |\tilde{R}| \equiv |\tilde{B}|/|\tilde{A}| = 1$ the complex longitudinal **specific** acoustic impedance $\tilde{z}_{a_{tot}}^{\parallel}(x=0)$ is **purely imaginary**; its phase $\varphi_z(x=0)$ is an **odd** integer multiple of $\pm\pi/2 = \pm 90^\circ$ – which in turn also tells us that in this situation, the complex pressure $\tilde{p}_{tot}(x=0, t)$ and longitudinal particle velocity $\tilde{u}_{tot}^{\parallel}(x=0, t)$ differ in phase by an **odd** integer multiple of $\pm\pi/2 = \pm 90^\circ$.

Note that in general, for arbitrary values of x , **maxima** of the complex longitudinal **specific** acoustic impedance $\tilde{z}_{a_{tot}}^{\parallel}(x)$ occur whenever $(2kx + \Delta\varphi_{BA}^o) = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi \dots = \pm n_{\text{even}} \pi$, *i.e.* whenever $\cos(2kx + \Delta\varphi_{BA}^o) = +1$, and thus $\sin(2kx + \Delta\varphi_{BA}^o) = 0$, then:

$$\begin{aligned} \frac{|\tilde{z}_{a_{tot}}^{\parallel}(x)|_{\text{maxima}}}{z_o} &= \frac{\sqrt{\left\{ 1 - |\tilde{R}|^2 \right\}^2 + 4|\tilde{R}|^2 \sin^2(2kx + \Delta\varphi_{BA}^o)}}{\left[\left\{ 1 + |\tilde{R}|^2 \right\} - 2|\tilde{R}| \cos(2kx + \Delta\varphi_{BA}^o) \right]} = \frac{\sqrt{\left\{ 1 - |\tilde{R}|^2 \right\}^2}}{\left[\left\{ 1 + |\tilde{R}|^2 \right\} - 2|\tilde{R}| \right]} \\ &= \left(\frac{1 - |\tilde{R}|^2}{1 - 2|\tilde{R}| + |\tilde{R}|^2} \right) = \left(\frac{1 - |\tilde{R}|^2}{(1 - |\tilde{R}|)^2} \right) = \frac{\cancel{(1 - |\tilde{R}|)} \cdot (1 + |\tilde{R}|)}{\cancel{(1 - |\tilde{R}|)} \cdot (1 - |\tilde{R}|)} = \frac{(1 + |\tilde{R}|)}{(1 - |\tilde{R}|)} = \frac{|\tilde{c}_{a_{tot}}^{\parallel}(x)|_{\text{maxima}}}{c} \end{aligned}$$