

Using the Euler relations: $\cos \theta \equiv \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ and: $\sin \theta \equiv \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$:

$$\tilde{z}_{a\,tot}^{\parallel}(x) = z_o \frac{\left[\left\{ 1 - |\tilde{R}|^2 \right\} + 2i|\tilde{R}|\sin(2kx + \Delta\phi_{BA}^o) \right]}{\left[\left\{ 1 + |\tilde{R}|^2 \right\} - 2|\tilde{R}|\cos(2kx + \Delta\phi_{BA}^o) \right]}$$

We can also write this as a dimensionless quantity:

$$\frac{\tilde{z}_{a\,tot}^{\parallel}(x)}{z_o} = \frac{\left[\left\{ 1 - |\tilde{R}|^2 \right\} + 2i|\tilde{R}|\sin(2kx + \Delta\phi_{BA}^o) \right]}{\left[\left\{ 1 + |\tilde{R}|^2 \right\} - 2|\tilde{R}|\cos(2kx + \Delta\phi_{BA}^o) \right]} = \frac{\tilde{c}_{a\,tot}^{\parallel}(x)}{c}$$

Note that $\tilde{z}_{a\,tot}^{\parallel}(x)$ and $\tilde{c}_{a\,tot}^{\parallel}(x)$ have **no** explicit time dependence, but both have spatial/position (x -) and frequency (f -) dependence (via the wavenumber $k = 2\pi/\lambda = 2\pi f/c = \omega/c$)!

The **magnitude** of the complex longitudinal specific acoustic impedance associated with the two counter-propagating 1-D monochromatic plane waves is:

$$\begin{aligned} \left| \tilde{z}_{a\,tot}^{\parallel}(x) \right| &\equiv \sqrt{\tilde{z}_{a\,tot}^{\parallel}(x) \cdot \tilde{z}_{a\,tot}^{\parallel*}(x)} \\ &= z_o \frac{\sqrt{\left[\left\{ 1 - |\tilde{R}|^2 \right\} + 2i|\tilde{R}|\sin(2kx + \Delta\phi_{BA}^o) \right] \cdot \left[\left\{ 1 - |\tilde{R}|^2 \right\} + 2i|\tilde{R}|\sin(2kx + \Delta\phi_{BA}^o) \right]^*}}{\left[\left\{ 1 + |\tilde{R}|^2 \right\} - 2|\tilde{R}|\cos(2kx + \Delta\phi_{BA}^o) \right]} \\ &= z_o \frac{\sqrt{\left[\left\{ 1 - |\tilde{R}|^2 \right\} + 2i|\tilde{R}|\sin(2kx + \Delta\phi_{BA}^o) \right] \cdot \left[\left\{ 1 - |\tilde{R}|^2 \right\} - 2i|\tilde{R}|\sin(2kx + \Delta\phi_{BA}^o) \right]}}{\left[\left\{ 1 + |\tilde{R}|^2 \right\} - 2|\tilde{R}|\cos(2kx + \Delta\phi_{BA}^o) \right]} \\ &= z_o \frac{\sqrt{\left\{ 1 - |\tilde{R}|^2 \right\}^2 + 4|\tilde{R}|^2 \sin^2(2kx + \Delta\phi_{BA}^o)}}{\left[\left\{ 1 + |\tilde{R}|^2 \right\} - 2|\tilde{R}|\cos(2kx + \Delta\phi_{BA}^o) \right]} \end{aligned}$$

Again, we can write this as a dimensionless quantity:

$$\left| \tilde{z}_{a\,tot}^{\parallel}(x) \right| = \frac{\sqrt{\left\{ 1 - |\tilde{R}|^2 \right\}^2 + 4|\tilde{R}|^2 \sin^2(2kx + \Delta\phi_{BA}^o)}}{\left[\left\{ 1 + |\tilde{R}|^2 \right\} - 2|\tilde{R}|\cos(2kx + \Delta\phi_{BA}^o) \right]} = \frac{\left| \tilde{c}_{a\,tot}^{\parallel}(x) \right|}{c}$$