$$\begin{aligned}\vec{\nabla}\tilde{p}(\vec{r},t) &= \left[\left\{\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right|\right\}\cdot e^{i\varphi_p(\vec{r},\omega)} + \left|\tilde{p}_o(\vec{r},\omega)\right|\cdot\vec{\nabla}e^{i\varphi_p(\vec{r},\omega)}\right]\cdot e^{i\omega t}\\
&= \left[\frac{\left\{\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right|\right\}}{\left|\tilde{p}_o(\vec{r},\omega)\right|}\left|\tilde{p}_o(\vec{r},\omega)\right|\cdot e^{i\varphi_p(\vec{r},\omega)} + i\left|\tilde{p}_o(\vec{r},\omega)\right|\cdot\left\{\vec{\nabla}\varphi_p(\vec{r},\omega)\right\}e^{i\varphi_p(\vec{r},\omega)}\right]\cdot e^{i\omega t}\\
&= \left[\frac{\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right|}{\left|\tilde{p}_o(\vec{r},\omega)\right|} + i\vec{\nabla}\varphi_p(\vec{r},\omega)\right]\underbrace{\left|\tilde{p}_o(\vec{r},\omega)\right|\cdot e^{i\varphi_p(\vec{r},\omega)}\cdot e^{i\omega t}}_{=\tilde{p}(\vec{r},t)}\\
&= \left[\frac{\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right|}{\left|\tilde{p}_o(\vec{r},\omega)\right|} + i\vec{\nabla}\varphi_p(\vec{r},\omega)\right]\tilde{p}(\vec{r},t)\end{aligned}$$

The Euler equation for this "generic" 3-D monochromatic traveling wave is:

$$i\omega\cdot\tilde{\vec{u}}(\vec{r},t) = -\frac{1}{\rho_o}\left[\frac{\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right|}{\left|\tilde{p}_o(\vec{r},\omega)\right|} + i\vec{\nabla}\varphi_p(\vec{r},\omega)\right]\tilde{p}(\vec{r},t)$$

or:

$$\begin{aligned}\tilde{\vec{u}}(\vec{r},t) &= -\frac{1}{i\rho_o\omega}\left[\frac{\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right|}{\left|\tilde{p}_o(\vec{r},\omega)\right|} + i\vec{\nabla}\varphi_p(\vec{r},\omega)\right]\tilde{p}(\vec{r},t) = +\frac{i}{\rho_o\omega}\left[\frac{\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right|}{\left|\tilde{p}_o(\vec{r},\omega)\right|} + i\vec{\nabla}\varphi_p(\vec{r},\omega)\right]\tilde{p}(\vec{r},t)\\
&= +\frac{1}{\rho_o\omega}\left[i\frac{\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right|}{\left|\tilde{p}_o(\vec{r},\omega)\right|} - \vec{\nabla}\varphi_p(\vec{r},\omega)\right]\tilde{p}(\vec{r},t) = -\frac{1}{\rho_o\omega}\left[\vec{\nabla}\varphi_p(\vec{r},\omega) - i\frac{\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right|}{\left|\tilde{p}_o(\vec{r},\omega)\right|}\right]\tilde{p}(\vec{r},t)\end{aligned}$$

Thus, for a "generic" 3-D monochromatic traveling wave, the complex ***time*-*domain*** 3-D particle velocity $\tilde{\vec{u}}(\vec{r},t)$ is related to the complex ***time*-*domain*** over-pressure amplitude $\tilde{p}(\vec{r},t)$ via the {linearized} Euler equation relation:

$$\boxed{\tilde{\vec{u}}(\vec{r},t) = -\frac{1}{\rho_o\omega}\left[\vec{\nabla}\varphi_p(\vec{r},\omega) - i\frac{\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right|}{\left|\tilde{p}_o(\vec{r},\omega)\right|}\right]\tilde{p}(\vec{r},t)}$$

There are two different kinds of terms/contributions on the RHS of this equation. The first term, $-\vec{\nabla}\varphi_p(\vec{r},\omega)$ is the {negative of the} spatial gradient of the ***phase*** of the complex over-pressure amplitude – note that for this contribution, $\tilde{\vec{u}}(\vec{r},t)$ is ***in*-*phase*** with $\tilde{p}(\vec{r},t)$. The second term, $+i\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right|/\left|\tilde{p}_o(\vec{r},\omega)\right|$ is the {normalized/fractional} spatial gradient of the complex over-pressure ***amplitude*** – note that for this contribution, $\tilde{\vec{u}}(\vec{r},t)$ is ***90^o*-*out*-*of*-*phase*** with $\tilde{p}(\vec{r},t)$. Then *e.g.* for the specific case of a monochromatic 3-D traveling plane wave, $\varphi_p(\vec{r},\omega) = -\vec{k}\bullet\vec{r}$ and $\tilde{p}_o(\vec{r},\omega) = p_o \neq fcn(\vec{r},\omega)$, thus: $\vec{\nabla}\varphi_p(\vec{r},\omega) = -\vec{\nabla}(\vec{k}\bullet\vec{r}) = -\vec{k}$ and: $\vec{\nabla}\left|\tilde{p}_o(\vec{r},\omega)\right| = 0$, hence {here} $\tilde{\vec{u}}(\vec{r},t)$ is ***in*-*phase*** with $\tilde{p}(\vec{r},t)$ and using $\omega = ck$ we also see that: $\tilde{\vec{u}}(\vec{r},t) = \left(\tilde{p}(\vec{r},t)/\rho_o c\right)\hat{k}$.

©Professor Steven Errede, Department of Physics, University of Illinois at Urbana-Champaign, Illinois
2002 - 2017. All rights reserved.