Examples of Complex Sound Fields:

Example # 0: "Generic"3-D Monochromatic Traveling Wave:

Before we launch into discussing several specific examples of complex sound fields/sound propagation, it is useful/illuminating to first consider the more general case of a "generic" complex sound field associated with a 3-D monochromatic traveling wave. Again, we assume that we are working in the ***linear*** regime of "everyday" sound pressure levels $SPL \ll 134\,dB\ \left(|\tilde{p}| \ll 100\,Pa\right)$ and also can safely ignore/neglect any/all dissipative effects, such that the Euler equation for inviscid fluid flow is a valid/accurate description of the acoustical physics situation. Then:

The complex ***time-domain*** over-pressure amplitude $\tilde{p}(\vec{r},t)$ associated with a "generic" 3-D monochromatic traveling wave at the listener space-time point $(\vec{r},t)$ can be written as:

$$\tilde{p}(\vec{r},t) = \left|\tilde{p}_o(\vec{r},\omega)\right| e^{i\left(\omega t+\varphi_p(\vec{r},\omega)\right)} = \underbrace{\left|\tilde{p}_o(\vec{r},\omega)\right| \cdot e^{i\varphi_p(\vec{r},\omega)}}_{\equiv \tilde{p}(\vec{r},\omega)} \cdot e^{i\omega t} = \tilde{p}(\vec{r},\omega)\cdot e^{i\omega t}$$

where: $\tilde{p}(\vec{r},\omega) = \left|\tilde{p}_o(\vec{r},\omega)\right| \cdot e^{i\varphi_p(\vec{r},\omega)}$ is the corresponding complex ***frequency-domain*** over-pressure amplitude associated with the "generic" 3-D monochromatic traveling wave at the listener space-time point $(\vec{r},t)$. Note that in general, both the magnitude of the complex over-pressure amplitude $\left|\tilde{p}_o(\vec{r},\omega)\right|$ and the phase $\varphi_p(\vec{r},\omega)$ are {listener} position-dependent and {angular} frequency-dependent quantities for a "generic" 3-D monochromatic traveling wave.

The {linearized} Euler equation for inviscid fluid flow (*i.e*. no dissipation) relates the complex ***time-domain*** 3-D particle velocity $\tilde{\vec{u}}(\vec{r},t)$ to the complex ***time-domain*** over-pressure amplitude $\tilde{p}(\vec{r},t)$:

$$\frac{\partial \tilde{\vec{u}}(\vec{r},t)}{\partial t} = -\frac{1}{\rho_o}\vec{\nabla}\tilde{p}(\vec{r},t)$$

In general, for "generic" 3-D monochromatic traveling wave, the complex ***time-domain*** 3-D particle velocity $\tilde{\vec{u}}(\vec{r},t)$ will be of the form: $\tilde{\vec{u}}(\vec{r},t) = \tilde{\vec{u}}(\vec{r},\omega)\cdot e^{i\omega t}$ where $\tilde{\vec{u}}(\vec{r},\omega)$ is the corresponding complex ***frequency-domain*** 3-D particle velocity.

On the LHS of the Euler equation, for a harmonic (*i.e*. monochromatic) complex sound field, since $\tilde{\vec{u}}(\vec{r},t) \propto e^{i\omega t}$, it is easy to show that $\partial\tilde{\vec{u}}(\vec{r},t)/\partial t = i\omega\tilde{\vec{u}}(\vec{r},t)$. Then on the RHS of the Euler equation:

$$\vec{\nabla}\tilde{p}(\vec{r},t) = \vec{\nabla}\tilde{p}(\vec{r},\omega)\cdot e^{i\omega t} = \vec{\nabla}\left[\left|\tilde{p}_o(\vec{r},\omega)\right| \cdot e^{i\varphi_p(\vec{r},\omega)}\right]\cdot e^{i\omega t}$$

Using the chain rule of differentiation, this relation becomes: