

Furthermore, if the nature of incident sound wave is such as to cause the air molecules within the infinitesimal volume element  $V_o$  to collectively move in a given direction, *i.e.* to be displaced by a collective 3-D distance  $\vec{\xi}(\vec{r}, t)$  from its equilibrium position, with collective velocity  $\vec{u}(\vec{r}, t)$  and collective acceleration  $\vec{a}(\vec{r}, t)$ , the “magical” Gaussian surface  $S_o$  co-moves with the air contained within  $V_o$ .

An infinitesimal volume element of size *e.g.* a cubic micron  $V_o = 1(\mu m)^3$  is statistically large enough for our purposes. The air contained within this infinitesimal volume element  $V_o$  is in thermal equilibrium with itself and with the air surrounding it. Avogadro’s number  $N_A = 6.022 \times 10^{23}$  *molecules/mole* and recall that one mole of {bone-dry} air @ NTP has mean/average molar mass of  $m_{mol}^{air} = 28.97$  *gms/mole*. Thus, for a volume mass density of air  $\rho_o = 1.204$  *kg/m<sup>3</sup>* at NTP there are  $24.06$  *cm<sup>3</sup>/mole*, or  $\sim 25$  *billion* molecules of air per cubic micron at NTP. The average/mean velocity vector associated with the mean/average thermal energy  $\langle U_{th}(\vec{r}, t) \rangle$  of this number of air molecules contained within the infinitesimal volume element  $V_o$  is  $\langle \vec{u}_{mol}(\vec{r}, t) \rangle = 0$ , however the thermal energies  $\langle E_{mol}^{th} \rangle = \frac{3}{2} k_B T = \frac{1}{2} m |\vec{u}_{mol}|^2$  associated with individual air molecules contained within  $V_o$  may be such that individual molecules within  $V_o$  leave through the bound surface  $S_o$  via exiting through one of the top, bottom or side surfaces associated with  $S_o$ . However, one of the other “magical” properties endowed with the co-moving surface  $S_o$  associated with the air contained within the infinitesimal volume element  $V_o$  is that if an air molecule leaves (enters) the bounding surface  $S_o$  at a given point  $\vec{r}_{mol}$  on one side of the volume element with velocity vector  $\vec{u}_{mol}(\vec{r}_{mol}, t)$ , it instantaneously enters (leaves) the surface  $S_o$  again with velocity vector  $\vec{u}_{mol}(\vec{r}_{mol}^{conj}, t)$ , but on the other side of the volume element, at its conjugate point  $\vec{r}_{mol}^{conj}$  relative to the center point  $(\vec{r}, t)$  of the infinitesimal volume element,  $V_o$ . Thus the total air mass  $m_{air}$ , the average/mean linear momentum  $\langle \vec{P}_{air}(\vec{r}, t) \rangle$  and the average/mean thermal energy  $\langle U_{th}(\vec{r}, t) \rangle$  are all conserved by this “magical” property of the fictitious Gaussian surface  $S$  bounding the infinitesimal volume element  $V_o$ .

From Newton’s 2<sup>nd</sup> law of motion,  $\vec{F}_{net} = m\vec{a}$ , we can calculate the force(s) acting on the air within the infinitesimal volume element  $V$  due to an over-pressure amplitude  $p(\vec{r}, t)$ . The mass of air contained within the infinitesimal volume element  $V_o$  is  $m = \rho_o V_o$  (*kg*). Newton’s 2<sup>nd</sup> law tells us that  $\vec{F}_{net}(\vec{r}, t) = m\vec{a}(\vec{r}, t)$  or that:  $\vec{a}(\vec{r}, t) = \vec{F}_{net}(\vec{r}, t)/m = \vec{F}_{net}(\vec{r}, t)/\rho_o V_o$ . We define the {net} force per unit volume acting on the infinitesimal volume element as:  $\vec{f}_{net}(\vec{r}, t) \equiv \vec{F}_{net}(\vec{r}, t)/V_o$ . Thus the acceleration  $\vec{a}(\vec{r}, t) = \vec{f}_{net}(\vec{r}, t)/\rho_o$ .

Next, let us (initially) consider only the *x*-component of the net force due to an over-pressure  $p(\vec{r}, t)$  acting on the infinitesimal volume element  $V_o$  of air, as shown in a side view in the figure below: