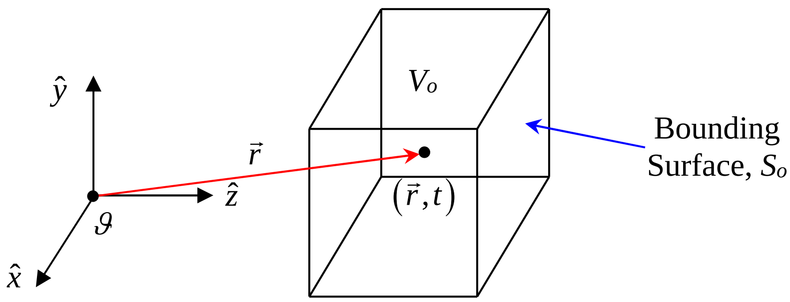
Derivation of Euler's Equation for Inviscid Fluid Flow from Newton's Second Law of Motion:

We can derive Euler's equation for inviscid fluid flow using Newton's 2nd law of motion $\left(\vec{F}_{net} = m\vec{a}\right)$ and at the same time gain some useful insight into the physical meaning of particle velocity, $\vec{u}\left(\vec{r},t\right)$.

Consider an infinitesimal volume element $V_o = 1\left(\mu m\right)^3$ bounded by the surface S_o centered on the space-time point $\left(\vec{r},t\right)$ {= center of mass of the infinitesimal volume element V_o} containing {bone-dry} air at NTP, in thermal equilibrium with the air surrounding it, and with equilibrium volume mass density $\rho_o = 1.204\, kg/m^3$, as shown in the figure below:



Rather than work in the fixed laboratory reference frame, we deliberately choose to work in a reference frame that is co-moving with the infinitesimal volume element V_o of air. Note that the pressure $p\left(\vec{r},t\right)$ associated with the infinitesimal volume element V_o as measured in the co-moving reference frame of the infinitesimal volume element V_o is the same pressure as measured in the fixed laboratory frame, this is because pressure $p\left(\vec{r},t\right)$ is intrinsically a scalar quantity.

The air {at NPT} contained within the infinitesimal volume element V_o is at a static / equilibrium absolute pressure of one atmosphere, *i.e.* $p_{atm} = 1.013\times10^5\, Pascals$ and a finite temperature $T = 20^o\, C$ (= 293.15 *K*). At the microscopic level, the air molecules within the infinitesimal volume element V_o each have mean thermal energy $\left\langle E_{mol}^{th}\right\rangle = \tfrac{3}{2}k_B T$ where $k_B = 1.381\times10^{-23}\, Joules/Kelvin$ and collide randomly with each other, undergoing Brownian random-walk type motions.

Suppose that a sound wave with over-pressure amplitude $\left|p\left(\vec{r},t\right)\right| \ll 100\, RMS\ Pascals$ { $SPL \ll 134\, dB$ } is incident on the {initially static} air contained within the infinitesimal volume element V_o. When the over-pressure amplitude $p\left(\vec{r},t\right)$ is instantaneously greater (less) than the ambient pressure p_{atm}, the air contained within V_o momentarily compresses (expands), respectively. Note that conceptually, the surface S_o that bounds the infinitesimal volume element V_o is endowed with "magical" properties, in that it is a fictitious, Gaussian-type surface (*e.g.* as commonly used in *E&M* problems), the nature of the bounding surface S_o also is one which expands and/or contracts as the air contained within the infinitesimal volume element V_o expands or contracts. Operationally this means we need only keep track of linear/leading-order terms in various expansions…