Exploiting the analog of the concept of electrical "voltage" – *i.e*. a difference in electrical potential $\Delta\Phi_e^{b\text{-}a} \equiv \Phi_e^b - \Phi_e^a = \int_a^b \vec{\nabla}\Phi_e(\vec{r})\cdot d\vec{\ell} = -\int_a^b \vec{E}(\vec{r})\cdot d\vec{\ell}$ we can also define a complex particle velocity potential ***difference*** (*aka* particle velocity "voltage") as:

$$\Delta\tilde{\Phi}_u^{b\text{-}a}(t) \equiv \tilde{\Phi}_u^b(t) - \Phi_u^a(t) = \int_a^b \vec{\nabla}\tilde{\Phi}_u(\vec{r},t)\cdot d\vec{\ell} = -\int_a^b \tilde{\vec{u}}(\vec{r},t)\cdot d\vec{\ell}$$

From the mass continuity equation: $\vec{\nabla}\bullet\tilde{\vec{u}}(\vec{r},t) = -\frac{1}{\rho_o}\left(\partial\tilde{\rho}(\vec{r},t)/\partial t\right)$ and: $\tilde{\vec{u}}(\vec{r},t) = \vec{\nabla}\tilde{\Phi}_u(\vec{r},t)$, then for "everyday" audio sound over-pressure amplitudes in {bone-dry} air at NTP of $\left|\tilde{p}(\vec{r},t)\right| \ll 100\ RMS\ Pascals$ $\{SPL \ll 134\, dB\}$, then: $\vec{\nabla}\bullet\vec{\nabla}\tilde{\Phi}_u(\vec{r},t) = -\frac{1}{\rho_o}\left(\partial\tilde{\rho}(\vec{r},t)/\partial t\right)$, which can be written as $\nabla^2\tilde{\Phi}_u(\vec{r},t) = -\frac{1}{\rho_o}\left(\partial\tilde{\rho}(\vec{r},t)/\partial t\right)$; this is Poisson's equation for the complex particle velocity potential!

Thus, we can thus solve {certain classes of} acoustical physics problems simply by solving Poisson's equation $\nabla^2\tilde{\Phi}_u(\vec{r},t) = -\frac{1}{\rho_o}\left(\partial\tilde{\rho}(\vec{r},t)/\partial t\right)$ for the complex particle velocity potential $\tilde{\Phi}_u(\vec{r},t)$, subject to the boundary condition(s) {and/or initial conditions at $t = 0$} associated with the specific problem using techniques/methodology similar to that used for solving Poisson's equation $\nabla^2\tilde{\Phi}_e(\vec{r}) \neq 0$ in E&M problems!

Note that {again} using the {linearized} adiabatic relationship between complex overpressure and mass density, $\tilde{\rho}(\vec{r},t) = \frac{1}{c^2}\tilde{p}(\vec{r},t)$ we also have: $\partial\tilde{\rho}(\vec{r},t)/\partial t \simeq \frac{1}{c^2}\partial\tilde{p}(\vec{r},t)/\partial t$. Hence for "everyday" audio sound fields, the above differential equation for the complex velocity potential can equivalently be written as: $\nabla^2\tilde{\Phi}_u(\vec{r},t) = -\frac{1}{\rho_o c^2}\left(\partial\tilde{p}(\vec{r},t)/\partial t\right)$.

If $\tilde{\vec{u}}(\vec{r},t) = \vec{\nabla}\tilde{\Phi}_u(\vec{r},t)$, the {linearized} Euler equation can be written as: $\frac{\partial\vec{\nabla}\tilde{\Phi}_u(\vec{r},t)}{\partial t} = \vec{\nabla}\frac{\partial\tilde{\Phi}_u(\vec{r},t)}{\partial t} \simeq -\frac{1}{\rho_o}\vec{\nabla}\tilde{p}(\vec{r},t)$, which implies that: $\frac{\partial\tilde{\Phi}_u(\vec{r},t)}{\partial t} \simeq -\frac{1}{\rho_o}\tilde{p}(\vec{r},t)$, and hence that: $\frac{\partial^2\tilde{\Phi}_u(\vec{r},t)}{\partial t^2} \simeq -\frac{1}{\rho_o}\frac{\partial\tilde{p}(\vec{r},t)}{\partial t}$. From above, we also have: $\frac{\partial\tilde{p}(\vec{r},t)}{\partial t} \simeq c^2\frac{\partial\tilde{\rho}(\vec{r},t)}{\partial t}$, thus: $\frac{\partial^2\tilde{\Phi}_u(\vec{r},t)}{\partial t^2} \simeq -\frac{c^2}{\rho_o}\frac{\partial\tilde{\rho}(\vec{r},t)}{\partial t}$, but from the above Poisson equation: $\nabla^2\tilde{\Phi}_u(\vec{r},t) = -\frac{1}{\rho_o}\frac{\partial\tilde{\rho}(\vec{r},t)}{\partial t}$, thus, we obtain the wave equation for the complex velocity potential:

$$\boxed{\nabla^2\tilde{\Phi}_u(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2\tilde{\Phi}_u(\vec{r},t)}{\partial t^2} = 0}$$