What is the ***curl*** of the 3-D particle velocity field, $\vec{\nabla}\times\vec{\tilde{u}}(\vec{r},t)=???$ Physically, the ***curl*** of a ***velocity*** field is often associated *e.g.* with a ***rotation*** and/or a velocity ***shear*** – such as the velocity field $\vec{v}(\vec{r},t)$ associated with a whirlpool, or a vortex in water. For this reason, the ***curl*** of a velocity field $\nabla\times\vec{v}(\vec{r},t)$ is sometimes known as/called the ***vorticity***.

However, in an ***inviscid*** fluid (*i.e.* one which is ***dissipationless***/has ***zero*** viscosity) such as air, the ***vorticity*** $\nabla\times\vec{v}(\vec{r},t)\equiv 0$, because an ***inviscid*** fluid ***cannot*** support velocity ***shears*** and/or ***vortices*** in the ***inviscid*** fluid. We can explicitly show/prove that $\vec{\nabla}\times\vec{\tilde{u}}(\vec{r},t)=0$ for "everyday" audio sound over-pressure amplitudes in air at NTP of $\left|\tilde{p}(\vec{r},t)\right|\ll 100\ RMS\ Pascals$. First, we take the partial derivative of $\vec{\nabla}\times\vec{\tilde{u}}(\vec{r},t)$ with respect to time:

$$\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{\tilde{u}}(\vec{r},t)\right)=\vec{\nabla}\times\frac{\partial\vec{\tilde{u}}(\vec{r},t)}{\partial t}$$

However, the Euler equation for inviscid fluid flow is: $\dfrac{\partial\vec{\tilde{u}}(\vec{r},t)}{\partial t}=-\dfrac{1}{\rho_o}\vec{\nabla}\tilde{p}(\vec{r},t)$, thus:

$$\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{\tilde{u}}(\vec{r},t)\right)=\vec{\nabla}\times\frac{\partial\vec{\tilde{u}}(\vec{r},t)}{\partial t}=-\frac{1}{\rho_o}\left(\vec{\nabla}\times\vec{\nabla}\tilde{p}(\vec{r},t)\right)$$

However, the ***curl*** of the ***gradient*** of any ***arbitrary*** scalar field $f(\vec{r},t)$ is also ***always*** zero, *i.e.* $\vec{\nabla}\times\vec{\nabla}f(\vec{r},t)\equiv 0$, thus:

$$\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{\tilde{u}}(\vec{r},t)\right)=\vec{\nabla}\times\frac{\partial\vec{\tilde{u}}(\vec{r},t)}{\partial t}=-\frac{1}{\rho_o}\left(\vec{\nabla}\times\vec{\nabla}\tilde{p}(\vec{r},t)\right)\equiv 0$$

This tells us that: $\vec{\nabla}\times\vec{\tilde{u}}(\vec{r},t)=constant\neq fcn(t)$. Thus, if for any time $-\infty\leq t\leq+\infty$, there is ***no*** vorticity in the inviscid fluid ($\vec{\nabla}\times\vec{\tilde{u}}(\vec{r},t)=0$), then it must ***remain*** = 0 for ***all*** time. *Q.E.D.*

If we take the time derivative of both sides of the {linearized} mass continuity equation, and the divergence of both sides of the {linearized} Euler equation:

$$\vec{\nabla}\bullet\frac{\partial\vec{\tilde{u}}(\vec{r},t)}{\partial t}\simeq-\frac{1}{\rho_o}\frac{\partial^2\tilde{\rho}(\vec{r},t)}{\partial t^2}=-\frac{1}{\rho_o c^2}\frac{\partial^2\tilde{p}(\vec{r},t)}{\partial t^2}\ \text{ and: }\ \vec{\nabla}\bullet\frac{\partial\vec{\tilde{u}}(\vec{r},t)}{\partial t}\simeq-\frac{1}{\rho_o}\vec{\nabla}\bullet\vec{\nabla}\tilde{p}(\vec{r},t)=-\frac{1}{\rho_o}\nabla^2\tilde{p}(\vec{r},t)$$

and then using the {linearized} adiabatic relationship between complex overpressure, $\tilde{p}$ and mass density, $\tilde{\rho}(\vec{r},t)=\frac{1}{c^2}\tilde{p}(\vec{r},t)$, we also have the relation: $\partial\tilde{\rho}(\vec{r},t)/\partial t\simeq\frac{1}{c^2}\partial\tilde{p}(\vec{r},t)/\partial t$.
Hence, we obtain the {linearized} wave equation for complex overpressure:

$$\boxed{\nabla^2\tilde{p}(\vec{r},t)-\frac{1}{c^2}\frac{\partial^2\tilde{p}(\vec{r},t)}{\partial t^2}=0}$$