Hence, for "everyday" audio sound fields, the ***linearized*** ***mass*** ***continuity*** ***equation*** is:

$$\frac{\partial \tilde{\rho}(\vec{r},t)}{\partial t} + \rho_o \vec{\nabla} \bullet \vec{\tilde{u}}(\vec{r},t) \simeq 0$$

Note also that for "everyday" audio sound fields, the ***linearized*** complex ***acoustic*** ***mass*** ***current*** ***density*** is: $\vec{\tilde{J}}_a(\vec{r},t) \simeq \rho_o\, \vec{\tilde{u}}(\vec{r},t) \quad \left(kg/m^2\text{-}s\right)$.

Likewise, for "everyday" audio sound fields, the ***non-linear*** Euler equation can likewise be ***linearized***. For $\left|\tilde{\rho}_a(\vec{r},t)\right| \ll \rho_o$, with $\tilde{\rho}(\vec{r},t) = \rho_o + \tilde{\rho}_a(\vec{r},t)$ we first make the approximation:

$$\tilde{\rho}(\vec{r},t)\frac{D\vec{\tilde{u}}(\vec{r},t)}{Dt} \Rightarrow \rho_o \frac{D\vec{\tilde{u}}(\vec{r},t)}{Dt} = \rho_o \left(\frac{\partial \vec{\tilde{u}}(\vec{r},t)}{\partial t} + \left(\vec{\tilde{u}}(\vec{r},t) \bullet \vec{\nabla}\right)\vec{\tilde{u}}(\vec{r},t) \right)$$

A second approximation that we now make for "everyday" audio sound fields is that it can be shown that the magnitude of the ***non-linear*** term $\left(\vec{\tilde{u}}(\vec{r},t) \bullet \vec{\nabla}\right)\vec{\tilde{u}}(\vec{r},t)$ is very small in comparison to the magnitude of the $\partial \vec{\tilde{u}}(\vec{r},t)/\partial t$ term, and hence can be neglected. Thus, the ***linearized*** version of Euler's equation, valid for $SPL \ll 134\, dB$ (over-pressure amplitudes $\left|\tilde{p}(\vec{r},t)\right| \ll 100\ RMS\ Pascals$) becomes:

$$\rho_o \frac{\partial \vec{\tilde{u}}(\vec{r},t)}{\partial t} \simeq -\vec{\nabla}\tilde{p}(\vec{r},t) \quad \text{or:} \quad \frac{\partial \vec{\tilde{u}}(\vec{r},t)}{\partial t} \simeq -\frac{1}{\rho_o}\vec{\nabla}\tilde{p}(\vec{r},t)$$

The **Helmholtz** **Theorem** tells us that the vectorial nature of an ***arbitrary*** vector field $\vec{\tilde{F}}(\vec{r})$ is ***fully-specified/unique*** if $a.)$ $\lim_{r\to\infty} \vec{\tilde{F}}(\vec{r}) \to 0$ and $b.)$ the ***divergence*** **.and.** the ***curl*** of $\vec{\tilde{F}}(\vec{r})$ are ***both*** known, *i.e.* $\vec{\nabla} \bullet \vec{\tilde{F}}(\vec{r}) = \tilde{C}(\vec{r})$ and $\vec{\nabla} \times \vec{\tilde{F}}(\vec{r}) = \vec{\tilde{D}}(\vec{r})$, with the restriction that $\vec{\nabla} \bullet \left(\vec{\nabla} \times \vec{\tilde{F}}(\vec{r})\right) = \vec{\nabla} \bullet \vec{\tilde{D}}(\vec{r}) \equiv 0$, since the divergence of the curl of ***any*** vector field is ***always*** zero.

For the 3-D particle velocity $\vec{\tilde{u}}(\vec{r},t)$ associated with sound waves propagating in an inviscid fluid such as air, for "everyday" over-pressure amplitudes of $\left|\tilde{p}(\vec{r},t)\right| \ll 100\ RMS\ Pascals$, we showed above that the ***linearized*** mass continuity equation (expressing conservation of mass), tells us that the spatial ***divergence*** of the 3-D particle velocity field is equal to the negative of the normalized (*aka* fractional) time rate of change of the volume mass density:

$$\vec{\nabla} \bullet \vec{\tilde{u}}(\vec{r},t) \simeq -\frac{1}{\rho_o}\frac{\partial \tilde{\rho}(\vec{r},t)}{\partial t}$$