

For “everyday” harmonic/single-frequency sound fields, if the 3-D vector complex **frequency-domain** particle velocity amplitude  $\vec{u}(\vec{r}, \omega)$  is known/measured, then since the 3-D vector complex **time-domain** particle velocity  $\vec{u}(\vec{r}, t) = \vec{u}(\vec{r}, \omega) \cdot e^{i\omega t}$ , and the 3-D vector complex **time-domain** particle displacement  $\vec{\xi}(\vec{r}, t) = \vec{\xi}(\vec{r}, \omega) \cdot e^{i\omega t}$ , where:  $\vec{\xi}(\vec{r}, \omega)$  is the 3-D vector complex **frequency-domain** particle displacement amplitude, and since  $\vec{u}(\vec{r}, t) = \partial \vec{\xi}(\vec{r}, t) / \partial t$ , then:

$$\vec{\xi}(\vec{r}, t) = \int \vec{u}(\vec{r}, t) dt = \int \vec{u}(\vec{r}, \omega) \cdot e^{i\omega t} dt = \vec{u}(\vec{r}, \omega) \int e^{i\omega t} dt = \frac{1}{i\omega} \vec{u}(\vec{r}, \omega) \cdot e^{i\omega t}$$

But since:  $\vec{\xi}(\vec{r}, t) = \vec{\xi}(\vec{r}, \omega) \cdot e^{i\omega t}$ , we see that:

$$\vec{\xi}(\vec{r}, \omega) = \frac{1}{i\omega} \vec{u}(\vec{r}, \omega) = -i \frac{1}{\omega} \vec{u}(\vec{r}, \omega)$$

Likewise, since:  $\vec{a}(\vec{r}, t) = \frac{\partial \vec{u}(\vec{r}, t)}{\partial t} = \frac{\partial \vec{u}(\vec{r}, \omega) \cdot e^{i\omega t}}{\partial t} = \vec{u}(\vec{r}, \omega) \cdot \frac{\partial e^{i\omega t}}{\partial t} = i\omega \cdot \vec{u}(\vec{r}, \omega) \cdot e^{i\omega t}$

But since:  $\vec{a}(\vec{r}, t) = \vec{a}(\vec{r}, \omega) \cdot e^{i\omega t}$ , we also see that:

$$\vec{a}(\vec{r}, \omega) = i\omega \cdot \vec{u}(\vec{r}, \omega)$$