

A mathematical statement associated with the conservation of mass within an infinitesimal volume element V of air of equilibrium volume V_o is given by: $\rho V = \rho_o V_o = \text{constant}$.

Thus, the **volumetric strain** (relevant for sound propagation in air) is: $\delta V/V = -\delta\rho/\rho$

or: $\delta\rho|_{\rho=\rho_o} = -\rho_o (\delta V/V)$, hence to **first** order the over-pressure:

$$p = \delta P = P - P_o \approx \left. \frac{\partial P}{\partial \rho} \right|_{\rho=\rho_o} \delta\rho = -\rho_o \left. \frac{\partial P}{\partial \rho} \right|_{\rho=\rho_o} \frac{\delta V}{V} = -B \frac{\delta V}{V}$$

where $B = \rho_o \partial P/\partial \rho|_{\rho=\rho_o}$ is the **adiabatic bulk modulus** of air {e.g. @ NTP}.

However, for **adiabatic** changes, the absolute air pressure $P = \text{constant} \times \rho^\gamma$ and thus:

$B = \rho_o \partial P/\partial \rho|_{\rho=\rho_o} = \gamma P_o$, hence:

$$p = \delta P = \left. \frac{\partial P}{\partial \rho} \right|_{\rho=\rho_o} \delta\rho = -\rho_o \left. \frac{\partial P}{\partial \rho} \right|_{\rho=\rho_o} \frac{\delta V}{V} = -B \frac{\delta V}{V} = +B \frac{\delta\rho}{\rho} = \gamma P_o \left(\frac{\rho - \rho_o}{\rho_o} \right) = \gamma P_o \cdot s$$

The fractional change in volume mass density is known as the **condensation**: $s \equiv \frac{\delta\rho}{\rho} \approx \frac{(\rho - \rho_o)}{\rho_o}$

Thus, for “everyday” audio sound over-pressure amplitudes $|\tilde{p}(\vec{r}, t)| \ll 100 \text{ RMS Pascals}$ { $SPL \ll 134 \text{ dB}$ }, the response of air as a medium for sound propagation is very nearly **linear**.

This in turn implies that for “everyday” sound over-pressure amplitudes, the volume mass density of air at NTP is nearly **constant**, i.e. $|\tilde{\rho}(\vec{r}, t)| \approx \rho_o = 1.204 \text{ kg/m}^3$ {i.e. $|\tilde{s}(\vec{r}, t)| \approx 0$ }.

However, for “everyday” audio sound over-pressure amplitudes, with **small** pressure variations ($|\tilde{p}(\vec{r}, t)| \ll P_o$), since: $\tilde{\rho}(\vec{r}, t) = \rho_o + \tilde{\rho}_a(\vec{r}, t)$, thus: $\tilde{\rho}_a(\vec{r}, t) = \delta\tilde{\rho}(\vec{r}, t) = \tilde{\rho}(\vec{r}, t) - \rho_o$

($|\tilde{\rho}_a(\vec{r}, t)| \ll \rho_o$) is the {incremental} volume mass density “amplitude” associated with the presence of the acoustic sound field, the time derivatives $\partial\tilde{\rho}(\vec{r}, t)/\partial t = \partial\tilde{\rho}_a(\vec{r}, t)/\partial t \neq 0$ and $\partial\tilde{s}(\vec{r}, t)/\partial t \neq 0$.

However, for $|\tilde{\rho}_a(\vec{r}, t)| \ll \rho_o$, the **non-linear** $\vec{\nabla} \cdot (\tilde{\rho}(\vec{r}, t) \vec{u}(\vec{r}, t))$ term in the **mass continuity equation** can be **linearized**:

$$\begin{aligned} \vec{\nabla} \cdot (\tilde{\rho}(\vec{r}, t) \vec{u}(\vec{r}, t)) &= \vec{\nabla} \cdot (\{\rho_o + \tilde{\rho}_a(\vec{r}, t)\} \vec{u}(\vec{r}, t)) \\ &= \rho_o \vec{\nabla} \cdot \vec{u}(\vec{r}, t) + \underbrace{\vec{\nabla} \cdot (\tilde{\rho}_a(\vec{r}, t) \vec{u}(\vec{r}, t))}_{\text{neglect}} \approx \rho_o \vec{\nabla} \cdot \vec{u}(\vec{r}, t) \end{aligned}$$