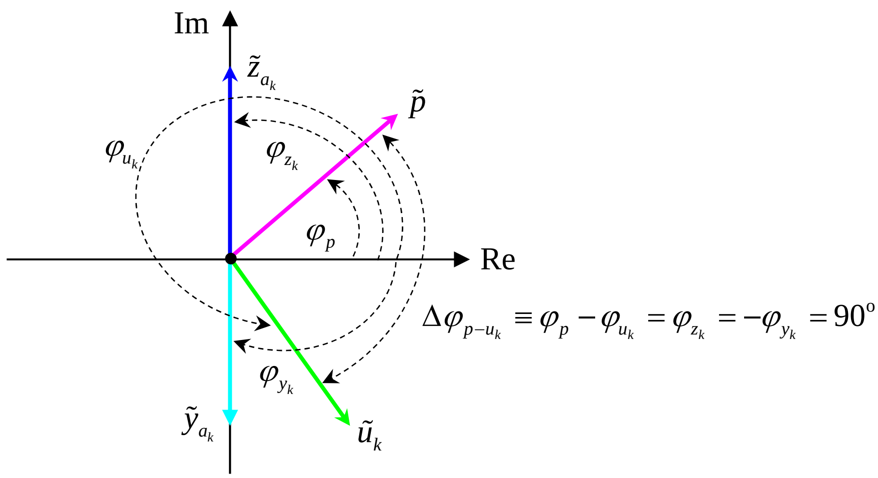
We also see that for harmonic/single-frequency sound fields the z_k -phase: $\varphi_{z_k} = \Delta\varphi_{p-u_k} \equiv \varphi_p - \varphi_{u_k}$ whereas the y_k -phase: $\varphi_{y_k} = \Delta\varphi_{u_k-p} \equiv \varphi_{u_k} - \varphi_p = -\left(\varphi_p - \varphi_{u_k}\right) = -\varphi_{z_k}$, in analogy to similar relations obtained *e.g.* for complex *AC* electrical circuits!

The phasor relation(s) in the complex plane for $\tilde{p} = p_{\mathrm{r}} + ip_{\mathrm{i}} = \left|\tilde{p}\right| e^{i\varphi_p}$, $\tilde{u}_k = u_{\mathrm{r}_k} + iu_{\mathrm{i}_k} = \left|\tilde{u}_k\right| e^{i\varphi_{u_k}}$, $\tilde{z}_{a_k} = z^{\mathrm{r}}_{a_k} + iz^{\mathrm{i}}_{a_k} = \left|\tilde{z}_{a_k}\right| e^{i\varphi_{z_k}}$ and $\tilde{y}_{a_k} = y^{\mathrm{r}}_{a_k} + iy^{\mathrm{i}}_{a_k} = \left|\tilde{y}_{a_k}\right| e^{i\varphi_{y_k}}$ are shown in the figure below, for the special/limiting case of $\Delta\varphi_{p-u_k} \equiv \varphi_p - \varphi_{u_k} = \varphi_{z_k} = -\varphi_{y_k} = 90^{\mathrm{o}}$, where the impedance phasor component $\tilde{z}_{a_k}$ is back-to-back with the admittance phasor component $\tilde{y}_{a_k}$ {*n.b.* for the more general case where $\Delta\varphi_{p-u_k} \equiv \varphi_p - \varphi_{u_k} = \varphi_{z_k} = -\varphi_{y_k} \neq 90^{\mathrm{o}}$, then $\tilde{z}_{a_k}$ and $\tilde{y}_{a_k}$ are ***not*** back-to-back}:



If we now take the ***cosine*** of the two phases φ_{z_k} and φ_{y_k} :

$$\cos\varphi_{z_k} = \cos\Delta\varphi_{p-u_k} \equiv \cos\left(\varphi_p - \varphi_{u_k}\right) \quad \text{and:}$$

$$\cos\varphi_{y_k} = \cos\Delta\varphi_{u_k-p} \equiv \cos\left(\varphi_{u_k} - \varphi_p\right) = \cos\left[-\varphi_{z_k}\right] = \cos\varphi_{z_k} \ (\cos(x) \text{ even fcn}(x))$$

We see that when: $\cos\varphi_{z_k} = \cos\varphi_{y_k} = +1$ that: $\Delta\varphi_{p-u_k} = -\Delta\varphi_{u_k-p} = 0^{\mathrm{o}}$, *i.e.* that: $\varphi_p = \varphi_{u_k}$.

When: $\cos\varphi_{z_k} = \cos\varphi_{y_k} = 0$ that: $\Delta\varphi_{p-u_k} = -\Delta\varphi_{u_k-p} = \pm 90^{\mathrm{o}}$, *i.e.* that: $\varphi_p = \varphi_{u_k} \pm 90^{\mathrm{o}}$.

When: $\cos\varphi_{z_k} = \cos\varphi_{y_k} = -1$ that: $\Delta\varphi_{p-u_k} = -\Delta\varphi_{u_k-p} = \pm 180^{\mathrm{o}}$, *i.e.* that: $\varphi_p = \varphi_{u_k} \pm 180^{\mathrm{o}}$.