We can gain further/additional insight into the nature of complex $\vec{\tilde{z}}_a(\vec{r},\omega)$ and $\vec{\tilde{y}}_a(\vec{r},\omega)$ by writing our primary acoustic ***frequency*-*domain*** variables in complex ***polar*** notation form:

Complex scalar pressure:

$$\tilde{p}(\vec{r},\omega) = p_{\mathrm{r}}(\vec{r},\omega) + ip_{\mathrm{i}}(\vec{r},\omega) = |\tilde{p}(\vec{r})| e^{i\varphi_p(\vec{r},\omega)}$$

Complex 3-D vector particle velocity:

$$\begin{aligned}\vec{\tilde{u}}(\vec{r},\omega) &= \vec{u}_{\mathrm{r}}(\vec{r},\omega) + i\vec{u}_{\mathrm{i}}(\vec{r},\omega)\\ &= \left[u_{\mathrm{r}_x}(\vec{r},\omega) + iu_{\mathrm{i}_x}(\vec{r},\omega)\right]\hat{x} + \left[u_{\mathrm{r}_y}(\vec{r},\omega) + iu_{\mathrm{i}_y}(\vec{r},\omega)\right]\hat{y} + \left[u_{\mathrm{r}_z}(\vec{r},\omega) + iu_{\mathrm{i}_z}(\vec{r},\omega)\right]\hat{z}\\ &= |\tilde{u}_x(\vec{r},\omega)| e^{i\varphi_{u_x}(\vec{r},\omega)}\hat{x} + |\tilde{u}_y(\vec{r},\omega)| e^{i\varphi_{u_y}(\vec{r},\omega)}\hat{y} + |\tilde{u}_z(\vec{r},\omega)| e^{i\varphi_{u_z}(\vec{r},\omega)}\hat{z}\end{aligned}$$

Complex 3-D vector *specific* acoustic admittance:

$$\begin{aligned}\vec{\tilde{y}}_a(\vec{r},\omega) &= \vec{y}_{\mathrm{r}}(\vec{r},\omega) + i\vec{y}_{\mathrm{i}}(\vec{r},\omega)\\ &= \left[y_{\mathrm{r}_x}(\vec{r},\omega) + iy_{\mathrm{i}_x}(\vec{r},\omega)\right]\hat{x} + \left[y_{\mathrm{r}_y}(\vec{r},\omega) + iy_{\mathrm{i}_y}(\vec{r},\omega)\right]\hat{y} + \left[y_{\mathrm{r}_z}(\vec{r},\omega) + iy_{\mathrm{i}_z}(\vec{r},\omega)\right]\hat{z}\\ &= |\tilde{y}_x(\vec{r},\omega)| e^{i\varphi_{y_x}(\vec{r},\omega)}\hat{x} + |\tilde{y}_y(\vec{r},\omega)| e^{i\varphi_{y_y}(\vec{r},\omega)}\hat{y} + |\tilde{y}_z(\vec{r},\omega)| e^{i\varphi_{y_z}(\vec{r},\omega)}\hat{z}\end{aligned}$$

Complex 3-D vector *specific* acoustic impedance:

$$\begin{aligned}\vec{\tilde{z}}_a(\vec{r},\omega) &= \vec{z}_{\mathrm{r}}(\vec{r},\omega) + i\vec{z}_{\mathrm{i}}(\vec{r},\omega)\\ &= \left[z_{\mathrm{r}_x}(\vec{r},\omega) + iz_{\mathrm{i}_x}(\vec{r},\omega)\right]\hat{x} + \left[z_{\mathrm{r}_y}(\vec{r},\omega) + iz_{\mathrm{i}_y}(\vec{r},\omega)\right]\hat{y} + \left[z_{\mathrm{r}_z}(\vec{r},\omega) + iz_{\mathrm{i}_z}(\vec{r},\omega)\right]\hat{z}\\ &= |\tilde{z}_x(\vec{r},\omega)| e^{i\varphi_{z_x}(\vec{r},\omega)}\hat{x} + |\tilde{z}_y(\vec{r},\omega)| e^{i\varphi_{z_y}(\vec{r},\omega)}\hat{y} + |\tilde{z}_z(\vec{r},\omega)| e^{i\varphi_{z_z}(\vec{r},\omega)}\hat{z}\end{aligned}$$

Thus, for harmonic/single-frequency sound fields we see that for a given $k = x, y,$ or z component of $\vec{\tilde{y}}_a(\vec{r},\omega)$, that:

$$\tilde{y}_{a_k}(\vec{r},\omega) = \frac{\tilde{u}_k(\vec{r},\omega)}{\tilde{p}(\vec{r},\omega)} \Rightarrow |\tilde{y}_{a_k}| e^{i\varphi_{y_k}} = \frac{|\tilde{u}_k| e^{i\varphi_{u_k}}}{|\tilde{p}| e^{i\varphi_p}} = \frac{|\tilde{u}_k|}{|\tilde{p}|} e^{-i\varphi_p} \cdot e^{i\varphi_{u_k}} = |\tilde{y}_{a_k}| e^{i\left[\varphi_{u_k} - \varphi_p\right]} = |\tilde{y}_{a_k}| e^{-i\Delta\varphi_{p-u_k}}$$

Similarly, for a given $k = x, y,$ or z component of $\vec{\tilde{z}}_a(\vec{r},\omega)$:

$$\tilde{z}_{a_k}(\vec{r},\omega) = \frac{\tilde{p}(\vec{r},\omega)\tilde{u}_k^*(\vec{r},\omega)}{|\tilde{u}(\vec{r},\omega)|^2} \Rightarrow |\tilde{z}_{a_k}| e^{i\varphi_{z_k}} = \frac{|\tilde{p}| e^{i\varphi_p} |\tilde{u}_k| e^{-i\varphi_{u_k}}}{|\vec{\tilde{u}}(\vec{r},\omega)|^2} = \frac{|\tilde{p}||\tilde{u}_k|}{|\vec{\tilde{u}}(\vec{r},\omega)|^2} e^{i\left[\varphi_p - \varphi_{u_k}\right]} = |\tilde{z}_{a_k}| e^{i\Delta\varphi_{p-u_k}}$$

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