

Thus, we see that for a given $k = x, y, \text{ or } z$ component of $\tilde{z}_a(\vec{r}, \omega)$:

$$\boxed{z_{a_k}^r = \operatorname{Re}\{\tilde{z}_{a_k}\} = \frac{P_r u_{r_k} + P_i u_{i_k}}{|\tilde{u}|^2}} \quad \text{and:} \quad \boxed{z_{a_k}^i = \operatorname{Im}\{\tilde{z}_{a_k}\} = \frac{P_i u_{r_k} - P_r u_{i_k}}{|\tilde{u}|^2}}$$

and we see that for a given $k = x, y, \text{ or } z$ component of $\tilde{y}_a(\vec{r}, \omega)$:

$$\boxed{y_{a_k}^r = \operatorname{Re}\{\tilde{y}_{a_k}\} = \frac{P_r u_{r_k} + P_i u_{i_k}}{|\tilde{p}|^2}} \quad \text{and:} \quad \boxed{y_{a_k}^i = \operatorname{Im}\{\tilde{y}_{a_k}\} = \frac{P_r u_{i_k} - P_i u_{r_k}}{|\tilde{p}|^2} = -\frac{P_i u_{r_k} - P_r u_{i_k}}{|\tilde{p}|^2}}$$

as well as: $\tilde{z}_{a_k} = \frac{\tilde{y}_{a_k}^*}{|\tilde{y}_a|^2}$ and: $\tilde{y}_{a_k} = \frac{\tilde{z}_{a_k}^*}{|\tilde{z}_a|^2}$ or equivalently: $\tilde{y}_{a_k} = |\tilde{y}_a|^2 \tilde{z}_{a_k}^*$ and: $\tilde{z}_{a_k} = |\tilde{z}_a|^2 \tilde{y}_{a_k}^*$.

It can be seen from these definitions that in **general** the individual vectorial components $k = x, y, \text{ or } z$ that: $\tilde{z}_{a_k}(\vec{r}, \omega)$ and $\tilde{y}_{a_k}(\vec{r}, \omega)$ do **not** point in the same direction in space.

Since $\tilde{z}_a(\vec{r}, \omega) = \tilde{p}(\vec{r}, \omega)/\tilde{u}(\vec{r}, \omega)$, another useful relation is: $\tilde{z}_a(\vec{r}, \omega) \cdot \tilde{u}(\vec{r}, \omega) = \tilde{p}(\vec{r}, \omega)$:

$$\begin{aligned} \tilde{z}_a(\vec{r}, \omega) \cdot \tilde{u}(\vec{r}, \omega) &= \left[\frac{\tilde{p}(\vec{r}, \omega)}{\tilde{u}(\vec{r}, \omega)} \right] \cdot \tilde{u}(\vec{r}, \omega) = \left[\frac{\tilde{p}(\vec{r}, \omega) \tilde{u}^*(\vec{r}, \omega)}{|\tilde{u}(\vec{r}, \omega)|^2} \right] \cdot \tilde{u}(\vec{r}, \omega) = \frac{\tilde{p}(\vec{r}, \omega) \cancel{|\tilde{u}(\vec{r}, \omega)|^2}}{\cancel{|\tilde{u}(\vec{r}, \omega)|^2}} \\ &= \tilde{p}(\vec{r}, \omega) \end{aligned}$$

Similarly, since $\tilde{y}_a(\vec{r}, \omega) = \tilde{u}(\vec{r}, \omega)/\tilde{p}(\vec{r}, \omega)$, then: $\tilde{y}_a(\vec{r}, \omega) \tilde{p}(\vec{r}, \omega) = \tilde{u}(\vec{r}, \omega)$.

Note that the above expressions for the real and imaginary components of complex acoustic specific impedance and/or admittance given in terms of linear combinations of the real and imaginary components of complex scalar acoustic over-pressure and complex vector particle velocity. As we have discussed previously, the physical meaning of the real and imaginary components of complex scalar acoustic over-pressure and complex vector particle velocity are respectively the in-phase and 90° (quadrature) components relative to the driving sound source. However, this is **not** the physical meaning of the real and imaginary components of complex acoustic specific immittances, because of the above-defined linear combinations of complex scalar acoustic over-pressure and complex vector particle velocity. We shall see/learn that the physical meaning of the real and imaginary components of complex acoustic immittances – properties of the physical medium in which acoustic disturbances propagate – are respectively associated with the **propagating** and **non-propagating** components of acoustic energy density.

The real and imaginary components of the acoustic specific immittances are often called the **active** and **reactive** components of the complex sound field, respectively, since (see above):

$$\boxed{\tilde{z}_a(\vec{r}, \omega) \equiv \vec{r}_a(\vec{r}, \omega) + i\vec{\chi}_a(\vec{r}, \omega) \quad (\Omega_a)} \quad \text{and:} \quad \boxed{\tilde{y}_a(\vec{r}, \omega) \equiv \vec{g}_a(\vec{r}, \omega) + i\vec{b}_a(\vec{r}, \omega) \quad (\Omega_a^{-1})}$$