

For harmonic/single-frequency sound fields, we can obtain expressions for the real and imaginary parts of frequency-domain complex 3-D vector ***specific*** acoustic impedance  $\tilde{z}_a(\vec{r}, \omega)$  and admittance  $\tilde{y}_a(\vec{r}, \omega)$  in terms of the real and imaginary parts of complex scalar over-pressure  $\tilde{p}(\vec{r}, \omega)$  and complex 3-D vector particle velocity  $\tilde{\vec{u}}(\vec{r}, \omega)$  from their respective definitions  $\tilde{z}_a(\vec{r}, \omega) = \tilde{p}(\vec{r}, \omega)/\tilde{\vec{u}}(\vec{r}, \omega)$  and  $\tilde{y}_a(\vec{r}, \omega) = \tilde{\vec{u}}(\vec{r}, \omega)/\tilde{p}(\vec{r}, \omega) = 1/\tilde{z}_a(\vec{r}, \omega)$ .

Suppressing the frequency-domain argument  $(\vec{r}, \omega)$  for notational clarity's sake, and working with only one of the three vectorial components  $k = x, y, \text{ or } z$ , for complex 3-D vector ***specific*** acoustic admittance:

$$\tilde{y}_{a_k} = y_{a_k}^r + iy_{a_k}^i = \frac{\tilde{u}_k}{\tilde{p}} = \frac{u_{r_k} + iu_{i_k}}{p_r + ip_i} = \left( \frac{u_{r_k} + iu_{i_k}}{p_r + ip_i} \right) \cdot \left( \frac{p_r - ip_i}{p_r - ip_i} \right) = \left( \frac{p_r u_{r_k} + p_i u_{i_k}}{|\tilde{p}|^2} \right) + i \left( \frac{p_r u_{i_k} - p_i u_{r_k}}{|\tilde{p}|^2} \right)$$

Thus we see that for  $k = x, y, \text{ or } z$ :

$$y_{a_k}^r = \text{Re}\{\tilde{y}_{a_k}\} = \frac{p_r u_{r_k} + p_i u_{i_k}}{|\tilde{p}|^2} \quad \text{and:} \quad y_{a_k}^i = \text{Im}\{\tilde{y}_{a_k}\} = \frac{p_r u_{i_k} - p_i u_{r_k}}{|\tilde{p}|^2} = -\frac{p_i u_{r_k} - p_r u_{i_k}}{|\tilde{p}|^2}$$

Likewise, for complex 3-D vector ***specific*** acoustic impedance:

$$\tilde{z}_{a_k} = z_{a_k}^r + iz_{a_k}^i = \frac{\tilde{p} \cdot \tilde{u}_k^*}{|\tilde{\vec{u}}|^2} = \frac{(p_r + ip_i)(u_{r_k} + iu_{i_k})^*}{|\tilde{\vec{u}}|^2} = \frac{(p_r + ip_i)(u_{r_k} - iu_{i_k})}{|\tilde{\vec{u}}|^2} = \left( \frac{p_r u_{r_k} + p_i u_{i_k}}{|\tilde{\vec{u}}|^2} \right) + i \left( \frac{p_i u_{r_k} - p_r u_{i_k}}{|\tilde{\vec{u}}|^2} \right)$$

Thus, we see that for  $k = x, y, \text{ or } z$ :

$$z_{a_k}^r = \text{Re}\{\tilde{z}_{a_k}\} = \frac{p_r u_{r_k} + p_i u_{i_k}}{|\tilde{\vec{u}}|^2} \quad \text{and:} \quad z_{a_k}^i = \text{Im}\{\tilde{z}_{a_k}\} = \frac{p_i u_{r_k} - p_r u_{i_k}}{|\tilde{\vec{u}}|^2}$$

Noting that:  $|\tilde{y}_{a_k}|^2 = \tilde{y}_{a_k} \cdot \tilde{y}_{a_k}^* = \frac{\tilde{u}_k \cdot \tilde{u}_k^*}{\tilde{p} \tilde{p}^*} = \frac{|\tilde{u}_k|^2}{|\tilde{p}|^2}$  and that:  $|\tilde{z}_{a_k}|^2 = \tilde{z}_{a_k} \cdot \tilde{z}_{a_k}^* = \frac{\tilde{p} \tilde{u}_k^* \cdot \tilde{p}^* \tilde{u}_k}{|\tilde{\vec{u}}|^2 |\tilde{\vec{u}}|^2} = \frac{|\tilde{p}|^2 |\tilde{u}_k|^2}{(|\tilde{\vec{u}}|^2)^2}$

We see that:  $|\tilde{\vec{u}}|^2 z_{a_k}^r = p_r u_{r_k} + p_i u_{i_k} = |\tilde{p}|^2 y_{a_k}^r$  and that:  $|\tilde{\vec{u}}|^2 z_{a_k}^i = p_i u_{r_k} - p_r u_{i_k} = -|\tilde{p}|^2 y_{a_k}^i$

or equivalently that:  $z_{a_k}^r = |\tilde{z}_a|^2 y_{a_k}^r$  or:  $y_{a_k}^r = |\tilde{y}_a|^2 z_{a_k}^r$  and that:  $z_{a_k}^i = -|\tilde{z}_a|^2 y_{a_k}^i$  or:  $y_{a_k}^i = -|\tilde{y}_a|^2 z_{a_k}^i$