Next, we explain why  $\vec{z}_a(\vec{r},\omega)$  and  $\vec{y}_a(\vec{r},\omega) = 1/\vec{z}_a(\vec{r},\omega)$  are called complex  $\Rightarrow$  <u>specific</u>  $\Leftarrow$ acoustic impedance and admittance, respectively. As mentioned above,  $\vec{z}_a(\vec{r},\omega)$  and  $\vec{y}_a(\vec{r},\omega) = 1/\vec{z}_a(\vec{r},\omega)$  are immittances <u>specifically</u> associated with the propagation medium. And, in order to avoid confusion, there {already} exists two <u>other</u> acoustic immittance quantities, known as the complex 3-D vector acoustic impedance  $\vec{Z}_a(\vec{r},\omega)$  and the complex 3-D vector acoustic admittance  $\vec{Y}_a(\vec{r},\omega) = 1/\vec{Z}_a(\vec{r},\omega)$ , which are associated with the acoustics of sound waves propagating inside <u>ducts</u> (*i.e.* pipes) with cross-sectional area  $S_{\perp}$  as defined below:

## **Complex 3-D Acoustic Immittances (for Harmonic Sound Fields):**

## **Complex 3-D Acoustic Impedance:**

$$\vec{\tilde{\mathbb{Z}}}_{a}(\vec{r},\omega) \equiv \frac{\tilde{p}(\vec{r},\omega)}{\vec{\tilde{u}}(\vec{r},\omega)S_{\perp}} = \frac{1}{\vec{\tilde{Y}}_{a}(\vec{r},\omega)} \begin{pmatrix} Pa-s/m^{3}\\ = N-s/m^{3} \end{pmatrix} = Rayl/m^{2}$$

## **Complex 3-D Acoustic Admittance:**

$$\vec{\tilde{Y}}_{a}(\vec{r},\omega) \equiv \frac{\vec{\tilde{u}}(\vec{r},\omega)S_{\perp}}{\tilde{p}(\vec{r},\omega)} = \frac{1}{\vec{\tilde{Z}}_{a}(\vec{r},\omega)} \begin{pmatrix} m/Pa-s \\ = m^{3}/N-s \end{pmatrix} = Rayl^{-1}-m^{2}$$

Note that the quantity  $\vec{U}(\vec{r},\omega) \equiv \vec{u}(\vec{r},\omega) S_{\perp}(m/s \cdot m^2 = m^3/s)$  is known as the <u>volume velocity</u>, because of its dimensions $(m^3/s)$ .

Inside a duct of cross sectional area  $S_{\perp}$ , the complex 3-D vector <u>specific</u> acoustic immittances  $\vec{z}_a(\vec{r},\omega)$  and  $\vec{y}_a(\vec{r},\omega) = 1/\vec{z}_a(\vec{r},\omega)$  are thus related to the complex 3-D vector immittances  $\vec{Z}_a(\vec{r},\omega)$  and  $\vec{Y}_a(\vec{r},\omega) = 1/\vec{Z}_a(\vec{r},\omega)$  by the relations:

$$\vec{\tilde{z}}_{a}(\vec{r},\omega) = \vec{\tilde{Z}}_{a}(\vec{r},\omega)S_{\perp} \quad \text{and} \quad \vec{\tilde{y}}_{a}(\vec{r},\omega) = \vec{\tilde{Y}}_{a}(\vec{r},\omega)/S_{\perp}$$
$$\vec{\tilde{Z}}_{a}(\vec{r},\omega) = \vec{\tilde{z}}_{a}(\vec{r},\omega)/S_{\perp} \quad \text{and} \quad \vec{\tilde{Y}}_{a}(\vec{r},\omega) = \vec{\tilde{y}}_{a}(\vec{r},\omega)S_{\perp}$$

or:

From the above relations, since the complex 3-D vector <u>specific</u> acoustic immittances  $\vec{z}_a(\vec{r},\omega)$  and  $\vec{y}_a(\vec{r},\omega) = 1/\vec{z}_a(\vec{r},\omega)$  are manifestly <u>frequency domain</u> quantities, we see that the complex 3-D vector acoustic immittances  $\vec{Z}_a(\vec{r},\omega)$  and  $\vec{Y}_a(\vec{r},\omega) = 1/\vec{Z}_a(\vec{r},\omega)$  are also manifestly <u>frequency domain</u> quantities.

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