Note also that *both* the *time-domain* complex pressure $\tilde{p}(\vec{r},t)$ and the *time-domain* complex 3-D particle velocity $\vec{u}(\vec{r},t)$ associated *e.g.* with a single frequency (*aka* harmonic) sound field will in general have time dependence of the form $e^{i\omega t}$. Thus, since the 3-D specific acoustic immittances are defined as *ratios* of these two quantities, the $e^{i\omega t}$ factor in the both the numerator and the denominator of the ratios $\vec{y}_a(\vec{r},t) = \frac{\vec{a}(\vec{r},t)}{\hat{p}(\vec{r},t)}$ and $\vec{\tilde{z}}_a(\vec{r},t) = \vec{p}(\vec{r},t)/\vec{\tilde{u}}(\vec{r},t)$ *cancels* for harmonic/single-frequency complex sound fields, thus we see that the complex 3-D vector specific acoustic immittances are in fact *time-independent* quantities… In fact, they are manifestly *frequency domain* quantities!

Time Domain:
$$
\vec{\tilde{y}}_a(\vec{r},t) = \frac{\vec{\tilde{u}}(\vec{r},t)}{\tilde{p}(\vec{r},t)} = \frac{\vec{\tilde{u}}(\vec{r},\omega) e^{j\omega t}}{\tilde{p}(\vec{r},\omega) e^{j\omega t}} = \frac{\vec{\tilde{u}}(\vec{r},\omega)}{\tilde{p}(\vec{r},\omega)} \equiv \vec{\tilde{y}}_a(\vec{r},\omega)
$$
 Frequency Domain

 $(\vec{r},t) = \frac{\tilde{p}(\vec{r},t)}{\vec{z}(\vec{r},t)}$ Time Domain: $\vec{\tilde{z}}_a(\vec{r},t) = \frac{\tilde{p}(\vec{r},t)}{\vec{\tilde{u}}(\vec{r},t)} = \frac{\tilde{p}(\vec{r},\omega) e^{j\omega t}}{\vec{\tilde{u}}(\vec{r},\omega) e^{j\omega t}}$ $\vec{z}_a(\vec{r},t) = \frac{\tilde{p}(\vec{r},t)}{\vec{z}(\vec{r},t)} = \frac{\tilde{p}(\vec{r},\omega) \hat{p}}{\vec{z}(\vec{r},t)}$ $\tilde{u}(\vec{r},t)$ $\equiv \frac{\tilde{p}(\vec{r},t)}{\vec{r}(\vec{r},t)} = \frac{\tilde{p}(\vec{r},\omega) e^{i\omega t}}{\vec{r}(\vec{r},t)}$ $\vec{\tilde{z}}_a(\vec{r},t) = \frac{\tilde{p}(\vec{r},t)}{\vec{\tilde{u}}(\vec{r},t)} = \frac{\tilde{p}(\vec{r},\omega) e^{i\omega t}}{\vec{\tilde{u}}(\vec{r},\omega) e^{i\omega t}}$ $\frac{\tilde{p}(\vec{r}, \omega)}{\tilde{a}(\vec{r}, \omega)} \equiv \tilde{z}_a(\vec{r}, \omega)$ Frequency Domain $\tilde{z}^{\vphantom{*}}_a(\vec{r}% _i^{\vphantom{*}})$ *u r* ω $=\frac{\tilde{p}(\vec{r},\omega)}{\vec{\tilde{u}}(\vec{r},\omega)}\equiv \vec{\tilde{z}}_{a}(\vec{r},\omega)$

Complex 3-D *Specific* **Acoustic Immittances (for Harmonic Sound Fields):**

Complex Specific Acoustic Impedance:

\n
$$
\overline{\vec{z}}_{a}(\vec{r}, \omega) = \frac{\tilde{p}(\vec{r}, \omega)}{\tilde{u}(\vec{r}, \omega)} = \frac{1}{\overline{\vec{y}}_{a}(\vec{r}, \omega)} \quad (\Omega_{a} = Rayl)
$$
\nTime-independent quantity!

\nComplex Specific Acoustic Admittance:

\n
$$
\overline{\vec{y}}_{a}(\vec{r}, \omega) = \frac{\vec{u}(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} = \frac{1}{\overline{\vec{z}}_{a}(\vec{r}, \omega)} \quad (\Omega_{a} = Rayl)
$$
\nTime-independent quantity!

\nThe independent quantity!

\nThe independent quantity!

\nTherefore, the following equation is:

\n
$$
\overline{\vec{y}}_{a}(\vec{r}, \omega) = \frac{\vec{u}(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} = \frac{1}{\overline{\vec{z}}_{a}(\vec{r}, \omega)} \quad (\Omega_{a}^{-1} = Rayl^{-1})
$$
\nTime-independent quantity!

\nTherefore, the following condition:

\n
$$
\overline{\vec{y}}_{a}(\vec{r}, \omega) = \frac{\vec{u}(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} = \frac{1}{\overline{\vec{z}}_{a}(\vec{r}, \omega)} \quad (\Omega_{a}^{-1} = Rayl^{-1})
$$

 The time-independent complex *specific* acoustic immittances are 3-D *vector frequencydomain* quantities. Their 3-D *x*-*y*-*z* Cartesian *frequency-domain* components can be explicitly written out as:

$$
\vec{y}_{a}(\vec{r},\omega) = \tilde{y}_{a_{x}}(\vec{r},\omega)\hat{x} + \tilde{y}_{a_{y}}(\vec{r},\omega)\hat{y} + \tilde{y}_{a_{z}}(\vec{r},\omega)\hat{z}
$$
\n
$$
= \frac{\tilde{u}_{x}(\vec{r},\omega)}{\tilde{p}(\vec{r},\omega)}\hat{x} + \frac{\tilde{u}_{y}(\vec{r},\omega)}{\tilde{p}(\vec{r},\omega)}\hat{y} + \frac{\tilde{u}_{z}(\vec{r},\omega)}{\tilde{p}(\vec{r},\omega)}\hat{z} = \frac{\tilde{u}(\vec{r},\omega)}{\tilde{p}(\vec{r},\omega)} = \frac{1}{\tilde{z}_{a}(\vec{r},\omega)}
$$
\n
$$
\vec{\tilde{z}}_{a}(\vec{r},\omega) = \tilde{z}_{a_{x}}(\vec{r},\omega)\hat{x} + \tilde{z}_{a_{y}}(\vec{r},\omega)\hat{y} + \tilde{z}_{a_{z}}(\vec{r},\omega)\hat{z} = \frac{\tilde{p}(\vec{r},\omega)}{\tilde{u}(\vec{r},\omega)} = \frac{1}{\tilde{y}_{a}(\vec{r},\omega)}
$$
\n
$$
= \frac{\tilde{p}(\vec{r},\omega)\tilde{u}_{x}^{*}(\vec{r},\omega)}{|\tilde{u}(\vec{r},\omega)|^{2}}\hat{x} + \frac{\tilde{p}(\vec{r},\omega)\tilde{u}_{y}^{*}(\vec{r},\omega)}{|\tilde{u}(\vec{r},\omega)|^{2}}\hat{y} + \frac{\tilde{p}(\vec{r},\omega)\tilde{u}_{z}^{*}(\vec{r},\omega)}{|\tilde{u}(\vec{r},\omega)|^{2}}\hat{z} = \frac{\tilde{p}(\vec{r},\omega)\tilde{u}^{*}(\vec{r},\omega)}{|\tilde{u}(\vec{r},\omega)|^{2}}
$$

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