

Note also that **both** the **time-domain** complex pressure $\tilde{p}(\vec{r}, t)$ and the **time-domain** complex 3-D particle velocity $\vec{\tilde{u}}(\vec{r}, t)$ associated e.g. with a single frequency (*aka* harmonic) sound field will in general have time dependence of the form $e^{i\omega t}$. Thus, since the 3-D specific acoustic immittances are defined as **ratios** of these two quantities, the $e^{i\omega t}$ factor in the both the numerator and the denominator of the ratios $\vec{\tilde{y}}_a(\vec{r}, t) = \vec{\tilde{u}}(\vec{r}, t)/\tilde{p}(\vec{r}, t)$ and $\vec{\tilde{z}}_a(\vec{r}, t) = \tilde{p}(\vec{r}, t)/\vec{\tilde{u}}(\vec{r}, t)$ **cancel**s for harmonic/single-frequency complex sound fields, thus we see that the complex 3-D vector specific acoustic immittances are in fact **time-independent** quantities... In fact, they are manifestly **frequency domain** quantities!

$$\text{Time Domain: } \vec{\tilde{y}}_a(\vec{r}, t) \equiv \frac{\vec{\tilde{u}}(\vec{r}, t)}{\tilde{p}(\vec{r}, t)} = \frac{\vec{\tilde{u}}(\vec{r}, \omega) e^{i\omega t}}{\tilde{p}(\vec{r}, \omega) e^{i\omega t}} = \frac{\vec{\tilde{u}}(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} \equiv \vec{\tilde{y}}_a(\vec{r}, \omega) \text{ Frequency Domain}$$

$$\text{Time Domain: } \vec{\tilde{z}}_a(\vec{r}, t) \equiv \frac{\tilde{p}(\vec{r}, t)}{\vec{\tilde{u}}(\vec{r}, t)} = \frac{\tilde{p}(\vec{r}, \omega) e^{i\omega t}}{\vec{\tilde{u}}(\vec{r}, \omega) e^{i\omega t}} = \frac{\tilde{p}(\vec{r}, \omega)}{\vec{\tilde{u}}(\vec{r}, \omega)} \equiv \vec{\tilde{z}}_a(\vec{r}, \omega) \text{ Frequency Domain}$$

Complex 3-D Specific Acoustic Immittances (for Harmonic Sound Fields):

$$\text{Complex Specific Acoustic Impedance: } \vec{\tilde{z}}_a(\vec{r}, \omega) \equiv \frac{\tilde{p}(\vec{r}, \omega)}{\vec{\tilde{u}}(\vec{r}, \omega)} = \frac{1}{\vec{\tilde{y}}_a(\vec{r}, \omega)} \quad (\Omega_a = \text{Rayl})$$

Time-independent quantity!
⇒ Frequency-domain quantity!

$$\text{Complex Specific Acoustic Admittance: } \vec{\tilde{y}}_a(\vec{r}, \omega) \equiv \frac{\vec{\tilde{u}}(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} = \frac{1}{\vec{\tilde{z}}_a(\vec{r}, \omega)} \quad (\Omega_a^{-1} = \text{Rayl}^{-1})$$

Time-independent quantity!
⇒ Frequency-domain quantity!

The time-independent complex **specific** acoustic immittances are 3-D **vector frequency-domain** quantities. Their 3-D *x-y-z* Cartesian **frequency-domain** components can be explicitly written out as:

$$\begin{aligned} \vec{\tilde{y}}_a(\vec{r}, \omega) &= \tilde{y}_{a_x}(\vec{r}, \omega) \hat{x} + \tilde{y}_{a_y}(\vec{r}, \omega) \hat{y} + \tilde{y}_{a_z}(\vec{r}, \omega) \hat{z} \\ &= \frac{\tilde{u}_x(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} \hat{x} + \frac{\tilde{u}_y(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} \hat{y} + \frac{\tilde{u}_z(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} \hat{z} = \frac{\vec{\tilde{u}}(\vec{r}, \omega)}{\tilde{p}(\vec{r}, \omega)} = \frac{1}{\vec{\tilde{z}}_a(\vec{r}, \omega)} \\ \vec{\tilde{z}}_a(\vec{r}, \omega) &= \tilde{z}_{a_x}(\vec{r}, \omega) \hat{x} + \tilde{z}_{a_y}(\vec{r}, \omega) \hat{y} + \tilde{z}_{a_z}(\vec{r}, \omega) \hat{z} = \frac{\tilde{p}(\vec{r}, \omega)}{\vec{\tilde{u}}(\vec{r}, \omega)} = \frac{1}{\vec{\tilde{y}}_a(\vec{r}, \omega)} \\ &= \frac{\tilde{p}(\vec{r}, \omega) \tilde{u}_x^*(\vec{r}, \omega)}{|\vec{\tilde{u}}(\vec{r}, \omega)|^2} \hat{x} + \frac{\tilde{p}(\vec{r}, \omega) \tilde{u}_y^*(\vec{r}, \omega)}{|\vec{\tilde{u}}(\vec{r}, \omega)|^2} \hat{y} + \frac{\tilde{p}(\vec{r}, \omega) \tilde{u}_z^*(\vec{r}, \omega)}{|\vec{\tilde{u}}(\vec{r}, \omega)|^2} \hat{z} = \frac{\tilde{p}(\vec{r}, \omega) \vec{\tilde{u}}^*(\vec{r}, \omega)}{|\vec{\tilde{u}}(\vec{r}, \omega)|^2} \end{aligned}$$