

$$\begin{aligned}
 |\tilde{z}_a(\vec{r}, t)| &= \sqrt{\tilde{z}_a(\vec{r}, t) \cdot \tilde{z}_a^*(\vec{r}, t)} = \sqrt{|\tilde{z}_{a_x}(\vec{r}, t)|^2 + |\tilde{z}_{a_y}(\vec{r}, t)|^2 + |\tilde{z}_{a_z}(\vec{r}, t)|^2} \\
 &= \sqrt{\frac{|\tilde{p}(\vec{r}, t)|^2 |\tilde{u}_x(\vec{r}, t)|^2}{\left(|\tilde{u}(\vec{r}, t)|^2\right)^2} + \frac{|\tilde{p}(\vec{r}, t)|^2 |\tilde{u}_y(\vec{r}, t)|^2}{\left(|\tilde{u}(\vec{r}, t)|^2\right)^2} + \frac{|\tilde{p}(\vec{r}, t)|^2 |\tilde{u}_z(\vec{r}, t)|^2}{\left(|\tilde{u}(\vec{r}, t)|^2\right)^2}} \\
 &= \sqrt{\frac{|\tilde{p}(\vec{r}, t)|^2 \left[|\tilde{u}_x(\vec{r}, t)|^2 + |\tilde{u}_y(\vec{r}, t)|^2 + |\tilde{u}_z(\vec{r}, t)|^2\right]}{\left(|\tilde{u}(\vec{r}, t)|^2\right)^2}} = \sqrt{\frac{|\tilde{p}(\vec{r}, t)|^2 |\tilde{u}(\vec{r}, t)|^2}{\left(|\tilde{u}(\vec{r}, t)|^2\right)^2}} \\
 &= \sqrt{\frac{|\tilde{p}(\vec{r}, t)|^2}{|\tilde{u}(\vec{r}, t)|^2}} = \frac{|\tilde{p}(\vec{r}, t)|}{|\tilde{u}(\vec{r}, t)|} = \frac{1}{|\tilde{y}_a(\vec{r}, t)|}
 \end{aligned}$$

However, we also see for the individual x, y, z components of the complex 3-D vector specific acoustic immittances that:

$$\begin{aligned}
 \left\{ \tilde{y}_{a_x}(\vec{r}, t) \equiv \frac{\tilde{u}_x(\vec{r}, t)}{\tilde{p}(\vec{r}, t)} \right\} &\neq \left\{ \frac{1}{\tilde{z}_{a_x}(\vec{r}, t)} \equiv \frac{|\tilde{u}(\vec{r}, t)|^2}{\tilde{p}(\vec{r}, t) \tilde{u}_x^*(\vec{r}, t)} \right\} \\
 \left\{ \tilde{y}_{a_y}(\vec{r}, t) \equiv \frac{\tilde{u}_y(\vec{r}, t)}{\tilde{p}(\vec{r}, t)} \right\} &\neq \left\{ \frac{1}{\tilde{z}_{a_y}(\vec{r}, t)} \equiv \frac{|\tilde{u}(\vec{r}, t)|^2}{\tilde{p}(\vec{r}, t) \tilde{u}_y^*(\vec{r}, t)} \right\} \\
 \left\{ \tilde{y}_{a_z}(\vec{r}, t) \equiv \frac{\tilde{u}_z(\vec{r}, t)}{\tilde{p}(\vec{r}, t)} \right\} &\neq \left\{ \frac{1}{\tilde{z}_{a_z}(\vec{r}, t)} \equiv \frac{|\tilde{u}(\vec{r}, t)|^2}{\tilde{p}(\vec{r}, t) \tilde{u}_z^*(\vec{r}, t)} \right\}
 \end{aligned}$$

Additionally, the expressions for the complex 3-D vector specific acoustic immittances:

$$\tilde{y}_a(\vec{r}, t) \equiv \frac{\tilde{u}(\vec{r}, t)}{\tilde{p}(\vec{r}, t)} = \tilde{y}_{a_x}(\vec{r}, t) \hat{x} + \tilde{y}_{a_y}(\vec{r}, t) \hat{y} + \tilde{y}_{a_z}(\vec{r}, t) \hat{z} = \frac{\tilde{u}_x(\vec{r}, t)}{\tilde{p}(\vec{r}, t)} \hat{x} + \frac{\tilde{u}_y(\vec{r}, t)}{\tilde{p}(\vec{r}, t)} \hat{y} + \frac{\tilde{u}_z(\vec{r}, t)}{\tilde{p}(\vec{r}, t)} \hat{z}$$

and:

$$\tilde{z}_a(\vec{r}, t) \equiv \frac{\tilde{p}(\vec{r}, t)}{\tilde{u}(\vec{r}, t)} = \tilde{z}_{a_x}(\vec{r}, t) \hat{x} + \tilde{z}_{a_y}(\vec{r}, t) \hat{y} + \tilde{z}_{a_z}(\vec{r}, t) \hat{z} = \frac{\tilde{p}(\vec{r}, t) \left[\tilde{u}_x^*(\vec{r}, t) \hat{x} + \tilde{u}_y^*(\vec{r}, t) \hat{y} + \tilde{u}_z^*(\vec{r}, t) \hat{z} \right]}{\left| \tilde{u}(\vec{r}, t) \right|^2}$$

can be seen to mathematically behave properly *e.g.* under arbitrary rotations of the local 3-D coordinate system, as well as for rotations of 3-D sound sources, and also for complex 3-D sound fields composed of *e.g.* an arbitrary superposition/linear combination of three mutually-orthogonal propagating monochromatic plane traveling waves – propagating in the $+\hat{x}$, $+\hat{y}$ and $+\hat{z}$ directions, with common scalar complex pressure, $\tilde{p}_{tot}(\vec{r}, t) = \tilde{p}_1(\vec{r}, t) + \tilde{p}_2(\vec{r}, t) + \tilde{p}_3(\vec{r}, t)$.