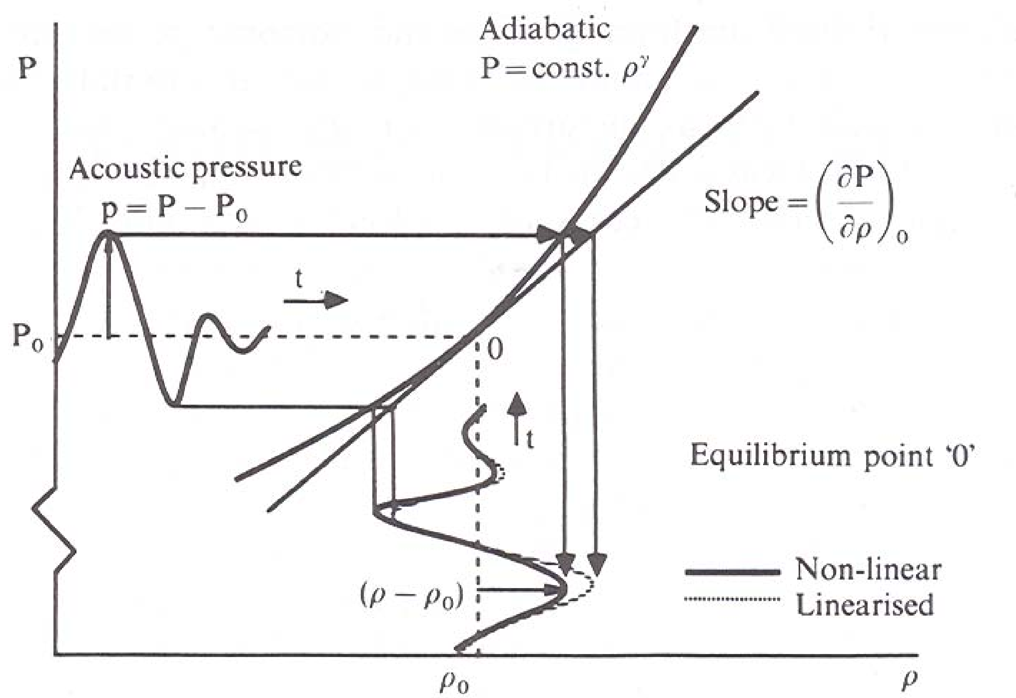
The ***non*-*linear*** response in air for ***large*** pressure variations $\left(SPL's \gtrsim 134\,dB\right)$ arises from the ***non*-*linear*** relation between the pressure and the density of air. For ***adiabatic*** changes in air pressure (relevant for sound propagation in air for audio frequency sounds {*i.e.* $f < 20\ KHz$}): $P(\vec{r},t) = P_{atm} + p(\vec{r},t) = constant \times \rho^{\gamma}(\vec{r},t)$ {where for air, $\gamma \equiv C_P/C_V \simeq 7/5 = 1.4$}. The relation between {absolute} pressure $P(\vec{r},t)$ and volume mass density $\rho^{\gamma}(\vec{r},t)$ of air is shown in the figure below, where equilibrium (*i.e.* no sound is present) $P_{atm} \equiv P_o$ and $\rho_{atm} \equiv \rho_o$:



We can express the instantaneous absolute pressure $P(\vec{r},t)$ as a Taylor series expansion about the equilibrium pressure $P_{atm} \equiv P_o$ and mass density $\rho_{atm} \equiv \rho_o$ configuration:

$$P(\vec{r},t) = P_o + \left.\frac{\partial P(\vec{r},t)}{\partial \rho(\vec{r},t)}\right|_{\rho=\rho_o} \left(\rho(\vec{r},t) - \rho_o\right) + \frac{1}{2}\left.\frac{\partial^2 P(\vec{r},t)}{\partial \rho^2(\vec{r},t)}\right|_{\rho=\rho_o} \left(\rho(\vec{r},t) - \rho_o\right)^2 + \ \ldots$$

$$= P_o + \left.\frac{\partial P(\vec{r},t)}{\partial \rho(\vec{r},t)}\right|_{\rho=\rho_o} \delta\rho(\vec{r},t) \quad + \quad \frac{1}{2}\left.\frac{\partial^2 P(\vec{r},t)}{\partial \rho^2(\vec{r},t)}\right|_{\rho=\rho_o} \left(\delta\rho(\vec{r},t)\right)^2 \quad + \ \ldots$$

For ***small*** pressure variations $\left(\left|\tilde{p}(\vec{r},t)\right| \ll P_{atm}\right)$ to ***first*** order, a ***linear*** relationship exists between over-pressure $p(\vec{r},t)$ and the volume mass density $\rho(\vec{r},t)$ for air:

$$p(\vec{r},t) = P(\vec{r},t) - P_o = \delta P(\vec{r},t) \simeq \left.\frac{\partial P(\vec{r},t)}{\partial \rho(\vec{r},t)}\right|_{\rho=\rho_o} \delta\rho(\vec{r},t)$$