Obviously, if we carry out *e.g*. a simple rotation of our local 3-D coordinate system, the individual *x*, *y*, *z* components of $\vec{\tilde{y}}_a(\vec{r},t)$ will change accordingly, however the magnitude $\left|\vec{\tilde{y}}_a(\vec{r},t)\right| = \sqrt{\vec{\tilde{y}}_a(\vec{r},t)\bullet\vec{\tilde{y}}_a^*(\vec{r},t)} = \sqrt{\left|\tilde{y}_{a_x}(\vec{r},t)\right|^2 + \left|\tilde{y}_{a_y}(\vec{r},t)\right|^2 + \left|\tilde{y}_{a_z}(\vec{r},t)\right|^2}$ will ***not*** change.

Likewise, the individual *x*, *y*, *z* components of $\vec{\tilde{z}}_a(\vec{r},t)$ will change accordingly under a simple rotation of our local 3-D coordinate system, however the magnitude $\left|\vec{\tilde{z}}_a(\vec{r},t)\right| = \sqrt{\vec{\tilde{z}}_a(\vec{r},t)\bullet\vec{\tilde{z}}_a^*(\vec{r},t)} = \sqrt{\left|\tilde{z}_{a_x}(\vec{r},t)\right|^2 + \left|\tilde{z}_{a_y}(\vec{r},t)\right|^2 + \left|\tilde{z}_{a_z}(\vec{r},t)\right|^2}$ will ***not*** change.

We thus write the complex 3-D vector ***specific*** acoustic impedance $\vec{\tilde{z}}_a(\vec{r},t)$, *e.g*. in Cartesian coordinates as follows:

$$\begin{aligned}
\vec{\tilde{z}}_a(\vec{r},t) &\equiv \frac{\tilde{p}(\vec{r},t)}{\vec{\tilde{u}}(\vec{r},t)} = \tilde{z}_{a_x}(\vec{r},t)\hat{x} + \tilde{z}_{a_y}(\vec{r},t)\hat{y} + \tilde{z}_{a_z}(\vec{r},t)\hat{z} \\
&= \frac{\tilde{p}(\vec{r},t)}{\vec{\tilde{u}}(\vec{r},t)}\bullet\frac{\vec{\tilde{u}}^*(\vec{r},t)}{\vec{\tilde{u}}^*(\vec{r},t)} = \frac{\tilde{p}(\vec{r},t)\vec{\tilde{u}}^*(\vec{r},t)}{\vec{\tilde{u}}(\vec{r},t)\bullet\vec{\tilde{u}}^*(\vec{r},t)} = \frac{\tilde{p}(\vec{r},t)\vec{\tilde{u}}^*(\vec{r},t)}{\left|\vec{\tilde{u}}(\vec{r},t)\right|^2} \\
&= \frac{\tilde{p}(\vec{r},t)\left[\tilde{u}_x^*(\vec{r},t)\hat{x} + \tilde{u}_y^*(\vec{r},t)\hat{y} + \tilde{u}_z^*(\vec{r},t)\hat{z}\right]}{\left|\vec{\tilde{u}}(\vec{r},t)\right|^2} \\
&= \frac{\tilde{p}(\vec{r},t)\left[\tilde{u}_x^*(\vec{r},t)\hat{x} + \tilde{u}_y^*(\vec{r},t)\hat{y} + \tilde{u}_z^*(\vec{r},t)\hat{z}\right]}{\left|\vec{\tilde{u}}_x(\vec{r},t)\right|^2 + \left|\vec{\tilde{u}}_y(\vec{r},t)\right|^2 + \left|\vec{\tilde{u}}_z(\vec{r},t)\right|^2}
\end{aligned}$$

where: $\tilde{z}_{a_x}(\vec{r},t) = \dfrac{\tilde{p}(\vec{r},t)\tilde{u}_x^*(\vec{r},t)}{\left|\vec{\tilde{u}}(\vec{r},t)\right|^2}$, $\tilde{z}_{a_y}(\vec{r},t) = \dfrac{\tilde{p}(\vec{r},t)\tilde{u}_y^*(\vec{r},t)}{\left|\vec{\tilde{u}}(\vec{r},t)\right|^2}$, $\tilde{z}_{a_z}(\vec{r},t) = \dfrac{\tilde{p}(\vec{r},t)\tilde{u}_z^*(\vec{r},t)}{\left|\vec{\tilde{u}}(\vec{r},t)\right|^2}$

Hence, the technical/mathematical issue here is the rationalization of an arbitrary, "generic" complex reciprocal 3-D vector quantity:

$$\vec{\tilde{u}}^{-1} = \frac{1}{\vec{\tilde{u}}} = \frac{\vec{\tilde{u}}^*}{\vec{\tilde{u}}\bullet\vec{\tilde{u}}^*} = \frac{\vec{\tilde{u}}^*}{\left|\vec{\tilde{u}}^*\right|^2}$$

paralleling that which is done for an arbitrary, "generic" complex reciprocal scalar quantity:

$$\tilde{p}^{-1} = \frac{1}{\tilde{p}} = \frac{\tilde{p}^*}{\tilde{p}\bullet\tilde{p}^*} = \frac{\tilde{p}^*}{\left|\tilde{p}^*\right|^2}$$

It can be seen that indeed: $\left|\vec{\tilde{y}}_a(\vec{r},t)\right| = \left|\dfrac{\vec{\tilde{u}}(\vec{r},t)}{\tilde{p}(\vec{r},t)}\right| = \dfrac{\left|\vec{\tilde{u}}(\vec{r},t)\right|}{\left|\tilde{p}(\vec{r},t)\right|} = \dfrac{1}{\left|\vec{\tilde{z}}_a(\vec{r},t)\right|}$, and also that: