<u>Complex 3-D Vector *Specific* Acoustic Immittances</u>:

<u>Cmplx *Spec.* Acoust. Impedance</u>: $\vec{\tilde{z}}_a(\vec{r},t) \equiv \dfrac{\tilde{p}(\vec{r},t)}{\vec{\tilde{u}}(\vec{r},t)} = \dfrac{1}{\vec{\tilde{y}}_a(\vec{r},t)} \quad \left(\begin{matrix} Acoustic \\ Ohms \end{matrix} \equiv \begin{matrix} Pa\text{-}s/m \\ = N\text{-}s/m^3 \end{matrix} = Rayl \right)$

<u>Cmplx *Spec.* Acoust. Admittance</u>: $\vec{\tilde{y}}_a(\vec{r},t) \equiv \dfrac{\vec{\tilde{u}}(\vec{r},t)}{\tilde{p}(\vec{r},t)} = \dfrac{1}{\vec{\tilde{z}}_a(\vec{r},t)} \quad \left(\begin{matrix} Acoustic \\ Siemens \end{matrix} \equiv \begin{matrix} m/Pa\text{-}s \\ = m^3/N\text{-}s \end{matrix} = Rayl^{-1} \right)$

Note that the complex ***<u>specific</u>*** acoustic immittances $\vec{\tilde{z}}_a(\vec{r},t)$ and $\vec{\tilde{y}}_a(\vec{r},t) = 1/\vec{\tilde{z}}_a(\vec{r},t)$ are 3-D ***<u>vector</u>*** quantities.

The complex 3-D vector ***<u>specific</u>*** acoustic ***<u>admittance</u>*** $\vec{\tilde{y}}_a(\vec{r},t) \equiv \vec{\tilde{u}}(\vec{r},t)/\tilde{p}(\vec{r},t)$ is clearly a mathematically well-defined vector quantity:

$$\vec{\tilde{y}}_a(\vec{r},t) \equiv \frac{\vec{\tilde{u}}(\vec{r},t)}{\tilde{p}(\vec{r},t)} = \tilde{y}_{a_x}(\vec{r},t)\hat{x} + \tilde{y}_{a_y}(\vec{r},t)\hat{y} + \tilde{y}_{a_z}(\vec{r},t)\hat{z} = \frac{\tilde{u}_x(\vec{r},t)}{\tilde{p}(\vec{r},t)}\hat{x} + \frac{\tilde{u}_y(\vec{r},t)}{\tilde{p}(\vec{r},t)}\hat{y} + \frac{\tilde{u}_z(\vec{r},t)}{\tilde{p}(\vec{r},t)}\hat{z}$$
$$= \frac{\left[\tilde{u}_x(\vec{r},t)\hat{x} + \tilde{u}_y(\vec{r},t)\hat{y} + \tilde{u}_z(\vec{r},t)\hat{z}\right]}{\tilde{p}(\vec{r},t)} = \frac{\vec{\tilde{u}}(\vec{r},t)}{\tilde{p}(\vec{r},t)}$$

$$\text{where: } \tilde{y}_{a_x}(\vec{r},t) = \frac{\tilde{u}_x(\vec{r},t)}{\tilde{p}(\vec{r},t)}, \quad \tilde{y}_{a_y}(\vec{r},t) = \frac{\tilde{u}_y(\vec{r},t)}{\tilde{p}(\vec{r},t)}, \quad \tilde{y}_{a_z}(\vec{r},t) = \frac{\tilde{u}_z(\vec{r},t)}{\tilde{p}(\vec{r},t)}$$

The complex 3-D vector ***<u>specific</u>*** acoustic impedance $\vec{\tilde{z}}_a(\vec{r},t) \equiv \tilde{p}(\vec{r},t)/\vec{\tilde{u}}(\vec{r},t)$ may initially seem like a mathematically less well-defined vector quantity. However, on physical/common sense grounds, we know that *e.g.* the ***<u>magnitudes</u>*** of the complex 3-D vector ***<u>specific</u>*** acoustic immittances, $\left|\vec{\tilde{y}}_a(\vec{r},t)\right|$ and $\left|\vec{\tilde{z}}_a(\vec{r},t)\right|$ must both be ***<u>invariant</u>*** (*i.e.* unchanged) under simple coordinate transformations – *e.g.* rotations and/or translations of the local coordinate system, as well as ***<u>invariant</u>*** under *e.g.* simple rotations of the sound source under investigation.

Consider a simple, 1-D monochromatic/single-frequency sound field – such as an acoustic traveling plane wave propagating *e.g.* in the local $+\hat{x}$ direction. Then $\tilde{u}_x(\vec{r},t) = u_o e^{i(\omega t - k_x x)} \neq 0$, with $\tilde{p}(\vec{r},t) = p_o e^{i(\omega t - k_x x)} \neq 0$, whereas $\tilde{u}_y(\vec{r},t) = \tilde{u}_z(\vec{r},t) = 0$. The components of the complex 3-D vector ***<u>specific</u>*** acoustic admittance are $\tilde{y}_{a_x}(\vec{r},t) = \tilde{u}_x(\vec{r},t)/\tilde{p}(\vec{r},t) = u_o \cancel{e^{i(\omega t - k_x x)}} / p_o \cancel{e^{i(\omega t - k_x x)}} = u_o/p_o \neq 0$, whereas $\tilde{y}_{a_y}(\vec{r},t) = \tilde{y}_{a_z}(\vec{r},t) = 0$.