Complex *Specific* Acoustic Immittances - Admittance and Impedance of a Medium:

The ***medium*** (solid, liquid or gas) in which sound waves propagate has associated with it the property of how easy (or how difficult) it is for sound waves to propagate through that medium – the so-called complex ***specific*** acoustic ***immittances*** – complex ***specific*** acoustic ***admittance*** and/or complex ***specific*** acoustic ***impedance*** (the reciprocal of complex ***specific*** acoustic ***admittance***) give us such information.

For propagation of 1-D sound waves in a medium, the complex ***specific*** acoustic immittances – *i.e*. collectively the complex ***specific*** acoustic admittance and/or complex ***specific*** acoustic impedance are both well-defined quantities. They are defined in analogy to the complex form of Ohm's Law ($\tilde{V}=\tilde{I}\tilde{Z}$, $\tilde{I}=\tilde{V}\tilde{Y}$) as used *e.g*. in electrical circuit theory, since complex over-pressure $\tilde{p}$ is the analog of complex *AC* voltage $\tilde{V}$, and particle velocity $\tilde{\vec{u}}$ is ~ the analog of complex *AC* electric current $\tilde{I}_e$ {Note that $\tilde{\vec{J}}_a(\vec{r},t)\equiv\rho_o\tilde{\vec{u}}(\vec{r},t)$ $\left(kg/s\text{-}m^2\right)$ is the complex acoustic ***mass*** current density}, whereas $\tilde{\vec{J}}_e\equiv\tilde{I}/\vec{A}_\perp=n_e q_e\tilde{\vec{v}}_e=\rho_e\tilde{\vec{v}}$ $\left(Amp/m^2=Coul/s\text{-}m^2\right)$ is the complex ***electrical*** current density}. Note also that both $\tilde{\vec{J}}_e$ and $\tilde{\vec{J}}_a$ are 3-D ***vector*** quantities.

Complex Scalar Electrical Immittances:

Complex Electrical Impedance: $$\boxed{\tilde{Z}_e(t;\omega)\equiv\frac{\tilde{V}(t;\omega)}{\tilde{I}_e(t;\omega)}\quad\left(Ohms=Volts/Amps\right)}$$

Complex Electrical Admittance: $$\boxed{\tilde{Y}_e(t;\omega)\equiv\frac{\tilde{I}_e(t;\omega)}{\tilde{V}(t;\omega)}\quad\left(Siemens=Ohms^{-1}=Amps/Volts\right)}$$

If we write out these relations using complex polar notation: $\tilde{V}(t;\omega)=\left|\tilde{V}(\omega)\right|e^{i\varphi_V(\omega)}\cdot e^{i\omega t}$, $\tilde{I}(t;\omega)=\left|\tilde{I}(\omega)\right|e^{i\varphi_I(\omega)}\cdot e^{i\omega t}$, then, noting the cancellation of $e^{i\omega t}$ time dependence factors:

$$\tilde{Z}_e(t;\omega)\equiv\frac{\tilde{V}(t;\omega)}{\tilde{I}_e(t;\omega)}=\frac{\left|\tilde{V}(\omega)\right|e^{i\varphi_V(\omega)}\cdot\cancel{e^{i\omega t}}}{\left|\tilde{I}_e(\omega)\right|e^{i\varphi_I(\omega)}\cdot\cancel{e^{i\omega t}}}=\frac{\left|\tilde{V}(\omega)\right|e^{i\varphi_V(\omega)}}{\left|\tilde{I}_e(\omega)\right|e^{i\varphi_I(\omega)}}=\frac{\left|\tilde{V}(\omega)\right|}{\left|\tilde{I}_e(\omega)\right|}e^{i[\varphi_V(\omega)-\varphi_I(\omega)]}=\left|\tilde{Z}_e(\omega)\right|e^{i\varphi_Z(\omega)}=\tilde{Z}_e(\omega)$$

$$\tilde{Y}_e(t;\omega)\equiv\frac{\tilde{I}_e(t;\omega)}{\tilde{V}(t;\omega)}=\frac{\left|\tilde{I}_e(\omega)\right|e^{i\varphi_I(\omega)}\cdot\cancel{e^{i\omega t}}}{\left|\tilde{V}(\omega)\right|e^{i\varphi_V(\omega)}\cdot\cancel{e^{i\omega t}}}=\frac{\left|\tilde{I}_e(\omega)\right|e^{i\varphi_I(\omega)}}{\left|\tilde{V}(\omega)\right|e^{i\varphi_V(\omega)}}=\frac{\left|\tilde{I}_e(\omega)\right|}{\left|\tilde{V}(\omega)\right|}e^{i[\varphi_I(\omega)-\varphi_V(\omega)]}=\left|\tilde{Y}_e(\omega)\right|e^{i\varphi_Y(\omega)}=\tilde{Y}_e(\omega)$$

Now: $\left|\tilde{Z}_e(\omega)\right|=1/\left|\tilde{Y}_e(\omega)\right|$ or: $\left|\tilde{Y}_e(\omega)\right|=1/\left|\tilde{Z}_e(\omega)\right|$, and we see that: $\varphi_Z(\omega)=\varphi_V(\omega)-\varphi_I(\omega)=-\varphi_Y(\omega)$, hence: $\tilde{Y}_e(\omega)=\left|\tilde{Y}_e(\omega)\right|e^{i\varphi_Y(\omega)}=\left\{1/\left|\tilde{Z}_e(\omega)\right|\right\}e^{-i\varphi_Z(\omega)}=1/\left\{\left|\tilde{Z}_e(\omega)\right|e^{i\varphi_Z(\omega)}\right\}=1/\tilde{Z}_e(\omega)$. Thus:

$$\boxed{\tilde{Z}_e(t;\omega)\equiv\frac{\tilde{V}(t;\omega)}{\tilde{I}_e(t;\omega)}=\frac{\tilde{V}(\omega)}{\tilde{I}_e(\omega)}=\tilde{Z}_e(\omega)=\frac{1}{\tilde{Y}_e(\omega)}}\quad\text{and:}\quad\boxed{\tilde{Y}_e(t;\omega)\equiv\frac{\tilde{I}_e(t;\omega)}{\tilde{V}(t;\omega)}=\frac{\tilde{I}_e(\omega)}{\tilde{V}(\omega)}=\tilde{Y}_e(\omega)=\frac{1}{\tilde{Z}_e(\omega)}}$$