

Note that the angular frequency “spikes” in the above figure at $\omega' = \omega$ associated with the real and imaginary components of the complex **frequency-domain** amplitude $\tilde{V}_{p-mic}(\vec{r}, \omega_o)$ are in fact 1-D **delta-functions** {in angular-frequency space}, which can be mathematically represented as $V_o^{p-mic}(\vec{r}, \omega_o) \cos \varphi_p(\vec{r}, \omega_o) \cdot \delta(\omega_o - \omega)$ and $V_o^{p-mic}(\vec{r}, \omega_o) \sin \varphi_p(\vec{r}, \omega_o) \cdot \delta(\omega_o - \omega)$, respectively. Note one of the many interesting/intriguing properties of the 1-D delta function: Since $\omega = 2\pi f$, hence $d\omega = 2\pi df$, and thus:

$$\begin{aligned} \int_{-\infty}^{+\infty} \delta(\omega_o - \omega) d\omega &= \int_{-\infty}^{+\infty} \delta(2\pi f_o - 2\pi f) \cdot 2\pi df = \int_{-\infty}^{+\infty} \delta[2\pi(f_o - f)] \cdot 2\pi df \\ &= \int_{-\infty}^{+\infty} \frac{1}{|2\pi|} \delta(f_o - f) \cdot 2\pi df = \int_{-\infty}^{+\infty} \frac{1}{2\pi} \delta(f_o - f) \cdot 2\pi df = \int_{-\infty}^{+\infty} \delta(f_o - f) df = 1 \end{aligned}$$

Note further that the 1-D delta function $\delta(\omega_o - \omega)$ has physical units of **inverse** angular frequency, $\omega^{-1} = 1/\omega$ (i.e. sec/radian) and that the 1-D delta function $\delta(f_o - f)$ has physical units of **inverse** frequency, $f^{-1} = 1/f$ (i.e. seconds), since the 1-D integrals $\int_{-\infty}^{+\infty} \delta(\omega_o - \omega) d\omega = 1$ and $\int_{-\infty}^{+\infty} \delta(f_o - f) df$ are both dimensionless...

The above complex **frequency-domain** result(s) should be compared with their complex **time-domain** counterparts:

$$\begin{aligned} \tilde{V}_{p-mic}(\vec{r}, t) &= V_o^{p-mic}(\vec{r}, \omega_o) e^{i(\omega_o t + \varphi_p(\vec{r}, \omega_o))} = V_o^{p-mic}(\vec{r}, \omega_o) e^{i\varphi_p(\vec{r}, \omega_o)} \cdot e^{i\omega_o t} \\ &= V_o^{p-mic}(\vec{r}, \omega_o) \left\{ \cos(\omega_o t + \varphi_p(\vec{r}, \omega_o)) + i \sin(\omega_o t + \varphi_p(\vec{r}, \omega_o)) \right\} \\ X(t) &\equiv \text{Re} \left\{ \tilde{V}_{p-mic}(\vec{r}, t) \right\} = \text{Re} \left\{ V_o^{p-mic}(\vec{r}, \omega_o) e^{i(\omega_o t + \varphi_p(\vec{r}, \omega_o))} \right\} = V_o^{p-mic}(\vec{r}, \omega_o) \text{Re} \left\{ e^{i(\omega_o t + \varphi_p(\vec{r}, \omega_o))} \right\} \\ &= V_o^{p-mic}(\vec{r}, \omega_o) \text{Re} \left\{ \cos(\omega_o t + \varphi_p(\vec{r}, \omega_o)) + i \cancel{\sin(\omega_o t + \varphi_p(\vec{r}, \omega_o))} \right\} \\ &= V_o^{p-mic}(\vec{r}, \omega_o) \cos(\omega_o t + \varphi_p(\vec{r}, \omega_o)) \\ Y(t) &\equiv \text{Im} \left\{ \tilde{V}_{p-mic}(\vec{r}, t) \right\} = \text{Im} \left\{ V_o^{p-mic}(\vec{r}, \omega_o) e^{i(\omega_o t + \varphi_p(\vec{r}, \omega_o))} \right\} = V_o^{p-mic}(\vec{r}, \omega_o) \text{Im} \left\{ e^{i(\omega_o t + \varphi_p(\vec{r}, \omega_o))} \right\} \\ &= V_o^{p-mic}(\vec{r}, \omega_o) \text{Im} \left\{ \cancel{\cos(\omega_o t + \varphi_p(\vec{r}, \omega_o))} + i \sin(\omega_o t + \varphi_p(\vec{r}, \omega_o)) \right\} \\ &= V_o^{p-mic}(\vec{r}, \omega_o) \sin(\omega_o t + \varphi_p(\vec{r}, \omega_o)) \end{aligned}$$

As mentioned above, the **frequency-domain** counterparts of complex **time-domain** quantities such as $\tilde{V}_{FG}(t) = V_o^{FG} e^{i\omega_o t}$ and $\tilde{V}_{p-mic}(\vec{r}, t) = V_o^{p-mic}(\vec{r}, \omega_o) e^{i(\omega_o t + \varphi_p(\vec{r}, \omega_o))}$ are obtained by taking the **Fourier transform** of the **time-domain** quantities, and vice-versa.