Note that the ***instantaneous time-domain*** voltage signals $V_{FG}(t)=V_o^{FG}\cos\omega_o t$ and $V_{p\text{-}mic}(\vec{r},t)=V_o^{p\text{-}mic}(\vec{r},\omega_o)\cos\left(\omega_o t+\varphi_p(\vec{r},\omega_o)\right)$ are ***purely real*** quantities. We can "***complexify***" these ***instantaneous time-domain*** quantities by adding ***quadrature/imaginary*** terms to them:

$$\tilde{V}_{FG}(t)=V_o^{FG}\cos\omega_o t+iV_o^{FG}\sin\omega_o t=V_o^{FG}\left(\cos\omega_o t+i\sin\omega_o t\right)=V_o^{FG}e^{i\omega_o t}\quad\text{and:}$$

$$\begin{aligned}\tilde{V}_{p\text{-}mic}(\vec{r},t)&=V_o^{p\text{-}mic}(\vec{r},\omega_o)\cos\left(\omega_o t+\varphi_p(\vec{r},\omega_o)\right)+iV_o^{p\text{-}mic}(\vec{r},\omega_o)\sin\left(\omega_o t+\varphi_p(\vec{r},\omega_o)\right)\\&=V_o^{p\text{-}mic}(\vec{r},\omega_o)\left\{\cos\left(\omega_o t+\varphi_p(\vec{r},\omega_o)\right)+i\sin\left(\omega_o t+\varphi_p(\vec{r},\omega_o)\right)\right\}=V_o^{p\text{-}mic}(\vec{r},\omega_o)e^{i\left(\omega_o t+\varphi_p(\vec{r},\omega_o)\right)}\end{aligned}$$

A {dual-channel} ***lock-in amplifier*** is ***<u>manifestly</u>*** a ***frequency-domain*** device that is routinely used in many types of physics experiments to simultaneously measure the real (*i.e.* in-phase) and imaginary/quadrature (*i.e.* 90° out-of-phase) components of a complex harmonic (*i.e.* single-frequency) signal, ***relative*** to a ***reference*** sine-wave signal of the same angular frequency $\omega_o=2\pi f_o$.

In the above example, we could *e.g.* additionally simultaneously connect the microphone's ***time-domain*** output signal $V_{p\text{-}mic}(\vec{r},t)=V_o^{p\text{-}mic}(\vec{r},\omega_o)\cos\left(\omega_o t+\varphi_p(\vec{r},\omega_o)\right)$ to the input of the lock-in amplifier and then ***<u>also</u>*** connect the TTL-level ***sync output*** of the sine-wave generator to the ***reference input*** of the lock-in amplifier, which is ***phase-locked*** to the actual instantaneous {***time-domain***} sine-wave voltage signal $V_{FG}(t)=V_o^{FG}\cos\omega_o t$ output from the sine-wave generator.

The lock-in amplifier then outputs ***frequency-domain*** real ("$X(\omega_o)$") and imaginary ("$Y(\omega_o)$") components of the complex *p*-mic signal that are respectively in-phase (90° out-of-phase) ***relative*** to the lock-in amplifier's ***reference input signal*** – in this case, the TTL-level ***sync signal*** output from the sine-wave generator:

$$\begin{aligned}X(\omega_o)&\equiv\operatorname{Re}\left\{\tilde{V}_{p\text{-}mic}(\vec{r},\omega_o)\right\}=\operatorname{Re}\left\{V_o^{p\text{-}mic}(\vec{r},\omega_o)e^{i\varphi_p(\vec{r},\omega_o)}\right\}=V_o^{p\text{-}mic}(\vec{r},\omega_o)\operatorname{Re}\left\{e^{i\varphi_p(\vec{r},\omega_o)}\right\}\\&=V_o^{p\text{-}mic}(\vec{r},\omega_o)\operatorname{Re}\left\{\cos\varphi_p(\vec{r},\omega_o)+i\,\cancel{\sin\varphi_p(\vec{r},\omega_o)}\right\}=V_o^{p\text{-}mic}(\vec{r},\omega_o)\cos\varphi_p(\vec{r},\omega_o)\\Y(\omega_o)&\equiv\operatorname{Im}\left\{\tilde{V}_{p\text{-}mic}(\vec{r},\omega_o)\right\}=\operatorname{Im}\left\{V_o^{p\text{-}mic}(\vec{r},\omega_o)e^{i\varphi_p(\vec{r},\omega_o)}\right\}=V_o^{p\text{-}mic}(\vec{r},\omega_o)\operatorname{Im}\left\{e^{i\varphi_p(\vec{r},\omega_o)}\right\}\\&=V_o^{p\text{-}mic}(\vec{r},\omega_o)\operatorname{Im}\left\{\cancel{\cos\varphi_p(\vec{r},\omega_o)}+i\sin\varphi_p(\vec{r},\omega_o)\right\}=V_o^{p\text{-}mic}(\vec{r},\omega_o)\sin\varphi_p(\vec{r},\omega_o)\end{aligned}$$

Thus, we see that the lock-in amplifier outputs the real (*i.e.* in-phase) and imaginary/quadrature {*i.e.* 90° out-of-phase) components of the ***frequency-domain*** complex voltage ***<u>amplitude</u>*** associated with the pressure microphone's output signal, obtained at the point $\vec{r}$ in the (complex) sound field of the loudspeaker:

$$\begin{aligned}\tilde{V}_{p\text{-}mic}(\vec{r},\omega_o)&=\operatorname{Re}\left\{\tilde{V}_{p\text{-}mic}(\vec{r},\omega_o)\right\}+i\operatorname{Im}\left\{\tilde{V}_{p\text{-}mic}(\vec{r},\omega_o)\right\}\\&=V_o^{p\text{-}mic}(\vec{r},\omega_o)\cos\varphi_p(\vec{r},\omega_o)+iV_o^{p\text{-}mic}(\vec{r},\omega_o)\sin\varphi_p(\vec{r},\omega_o)\\&=V_o^{p\text{-}mic}(\vec{r},\omega_o)\left\{\cos\varphi_p(\vec{r},\omega_o)+i\sin\varphi_p(\vec{r},\omega_o)\right\}=V_o^{p\text{-}mic}(\vec{r},\omega_o)e^{i\varphi_p(\vec{r},\omega_o)}\end{aligned}$$