The total ***net*** 3-D vector force per unit volume is therefore:

$$\vec{f}_{net}(\vec{r},t) = f_{net_x}(\vec{r},t)\hat{x} + f_{net_y}(\vec{r},t)\hat{y} + f_{net_z}(\vec{r},t)\hat{z}$$

$$= -\frac{\partial p(\vec{r},t)}{\partial x}\hat{x} - \frac{\partial p(\vec{r},t)}{\partial y}\hat{y} - \frac{\partial p(\vec{r},t)}{\partial y}\hat{z} = -\underbrace{\left(\frac{\partial}{\partial x}\hat{x} - \frac{\partial}{\partial y}\hat{y} - \frac{\partial}{\partial y}\hat{z}\right)}_{\equiv\vec{\nabla}} p(\vec{r},t) = -\vec{\nabla}p(\vec{r},t)$$

Thus we have: $\vec{a}(\vec{r},t) = \vec{f}(\vec{r},t)/\rho_o$ and: $\vec{f}_{net}(\vec{r},t) = \vec{f}(\vec{r},t) = -\vec{\nabla}p(\vec{r},t)$, hence: $\vec{a}(\vec{r},t) = -\vec{\nabla}p(\vec{r},t)/\rho_o$. Recall that (for $\left|p(\vec{r},t)\right| \ll 100\ RMS\ Pascals$ { $SPL \ll 134\ dB$ }, the particle acceleration $\vec{a}(\vec{r},t)$ is the time rate of change of the particle velocity $\vec{u}(\vec{r},t)$, *i.e.* $\vec{a}(\vec{r},t) = \partial\vec{u}(\vec{r},t)/\partial t$, hence we obtain Euler's equation for inviscid fluid flow, valid for air with $\left|p(\vec{r},t)\right| \ll 100\ RMS\ Pascals$ { $SPL \ll 134\ dB$ }:

$$\vec{a}(\vec{r},t) = \frac{\partial\vec{u}(\vec{r},t)}{\partial t} = -\frac{1}{\rho_o}\vec{\nabla}p(\vec{r},t) \quad Q.E.D.$$

"Complexifying" this equation, we have:

$$\boxed{\tilde{\vec{a}}(\vec{r},t) = \frac{\partial\tilde{\vec{u}}(\vec{r},t)}{\partial t} = -\frac{1}{\rho_o}\vec{\nabla}\tilde{p}(\vec{r},t)}$$

Although this relationship between the complex particle acceleration $\tilde{\vec{a}}(\vec{r},t)$, particle velocity $\tilde{\vec{u}}(\vec{r},t)$ and complex pressure $\tilde{p}(\vec{r},t)$ was derived in the co-moving/center-of-mass reference frame associated with the infinitesimal volume element V_o centered on the space-time point $(\vec{r},t)$, superimposed on top of a <u>static</u> pressure field $p_{atm} = 1.013\times10^5\ Pascals$, it can be seen that for <u>small</u>, <u>harmonic</u>/<u>periodic</u> over-pressure amplitude variations, *e.g.* $\tilde{p}(\vec{r},t) = \tilde{p}_o(\vec{r})e^{i\omega t}$ with $\left|\tilde{p}(\vec{r},t)\right| \ll p_{atm}$ that each of these quantities are the same in the laboratory reference frame.

We can now also see that the complex particle <u>displacement</u> $\tilde{\vec{\xi}}(\vec{r},t)$ (m) {from equilibrium position}, complex particle <u>velocity</u> $\tilde{\vec{u}}(\vec{r},t) = \partial\tilde{\vec{\xi}}(\vec{r},t)/\partial t$ (m/s) and complex particle <u>acceleration</u> $\tilde{\vec{a}}(\vec{r},t) = \partial\tilde{\vec{u}}(\vec{r},t)/\partial t$ (m/s^2) are associated with the <u>***collective***</u>, random-thermal energy-averaged-out motion of the air molecules contained within the infinitesimal volume element V_o bounded by the {co-moving} surface S_o centered on the space-time point $(\vec{r},t)$.