Thus, finally we see that:

$$\begin{aligned}\left|\tilde{Z}_{tot}\right|^2 &= \left|\tilde{Z}_1\right|^2 + \tilde{Z}_1\cdot\tilde{Z}_2^* + \tilde{Z}_2\cdot\tilde{Z}_1^* + \left|\tilde{Z}_2\right|^2 \\ &= \left|\tilde{Z}_1\right|^2 + \left|\tilde{Z}_2\right|^2 + \left(\tilde{Z}_1\cdot\tilde{Z}_2^* + \tilde{Z}_2\cdot\tilde{Z}_1^*\right) \\ &= \left|\tilde{Z}_1\right|^2 + \left|\tilde{Z}_2\right|^2 + 2\,\mathrm{Re}\left\{\tilde{Z}_1\cdot\tilde{Z}_2^*\right\}\end{aligned}$$

If we now insert the explicit expressions for complex $\tilde{Z}_1(\vec{r},t) = A_1(\vec{r},t)e^{i(\omega_1(t)t+\varphi_1(t))}$ and $\tilde{Z}_2(\vec{r},t) = A_2(\vec{r},t)e^{i(\omega_2(t)t+\varphi_2(t))}$ in the above formula:

$$\begin{aligned}\left|\tilde{Z}_{tot}\right|^2 &= A_1^2 + A_2^2 + 2A_1\cdot A_2\cos\left[(\omega_1 t+\varphi_1)-(\omega_2 t+\varphi_2)\right] \\ &= A_1^2 + A_2^2 + 2A_1\cdot A_2\cos\left[(\omega_1 t-\omega_2 t)+(\varphi_1-\varphi_2)\right] \\ &= A_1^2 + A_2^2 + 2A_1\cdot A_2\cos\left[(\omega_1-\omega_2)t+(\varphi_1-\varphi_2)\right]\end{aligned}$$

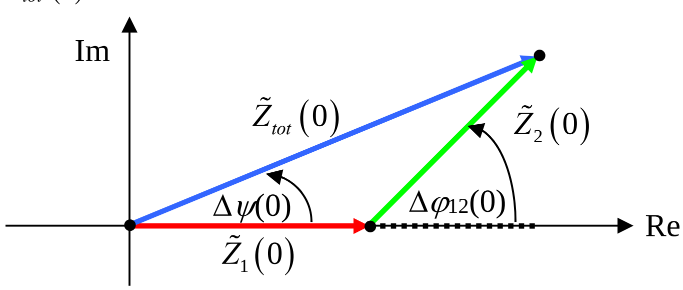
Let us now define $\Delta\omega_{12}(t)\equiv(\omega_1(t)-\omega_2(t))$ and $\Delta\varphi_{12}(\vec{r},t)\equiv(\varphi_1(\vec{r},t)-\varphi_2(\vec{r},t))$.
Then we see that:

$$\left|\tilde{Z}_{tot}\right|^2 = A_1^2 + A_2^2 + 2A_1\cdot A_2\cos\left[\Delta\omega_{12}t+\Delta\varphi_{12}\right]$$

If the frequencies of the two complex amplitudes are equal, then $\Delta\omega_{12}(t)\equiv(\omega_1(t)-\omega_2(t))=0$ and thus:

$$\left|\tilde{Z}_{tot}\right|^2 = A_1^2 + A_2^2 + 2A_1\cdot A_2\cos\Delta\varphi_{12}$$

Note that this expression is simply the formula for the **law of cosines** associated with a triangle lying in the complex plane! The *phasor diagram* associated with the two complex amplitudes $\tilde{Z}_1(\vec{r},t) = A_1(\vec{r},t)e^{i(\omega_1(t)t+\varphi_1(\vec{r},t))}$ and $\tilde{Z}_2(t) = A_2(t)e^{i(\omega_2(t)t+\varphi_2(\vec{r},t))}$ and their resulting overall amplitude $\tilde{Z}_{tot}(t)$ in the complex plane is shown in the figure below, for $t = 0$.



Note that as time t increases, the phasor triangle diagram rotates counter-clockwise in the complex plane – and also potentially in a quite complicated manner, *e.g.* if $\omega_1(t)\neq\omega_2(t)$.

For any complex quantity $\tilde{Z}(\vec{r},t) = X(\vec{r},t)+iY(\vec{r},t)$, the phase angle $\varphi(\vec{r},t)$ relative to the real axis (*i.e.* the *x*-axis) in the complex plane is given by the simple trigonometric formula: $\tan\varphi = Y/X$ or: $\varphi(t)=\tan^{-1}(Y/X)$.