Then (again, for clarity's sake, temporarily suppressing the $(\vec{r},t)$-dependence of these quantities):

$$\begin{aligned}\tilde{Z}_1\cdot\tilde{Z}_2^*+\tilde{Z}_2\cdot\tilde{Z}_1^* &= (X_1+iY_1)\cdot(X_2+iY_2)^*+(X_2+iY_2)\cdot(X_1+iY_1)^*\\ &=(X_1+iY_1)\cdot(X_2-iY_2)\ +(X_2+iY_2)\cdot(X_1-iY_1)\\ &=(X_1\cdot X_2+iY_1\cdot X_2-iY_2\cdot X_1+Y_1\cdot Y_2)\ +\ (X_2\cdot X_1+iY_2\cdot X_1-iY_1\cdot X_2+Y_2\cdot Y_1)\\ &=\left(X_1\cdot X_2+i\,\cancel{X_2\cdot Y_1}-i\,\cancel{X_1\cdot Y_2}+Y\cdot_1 Y_2\right)+\left(X_1\cdot X_2+i\,\cancel{X_1\cdot Y_2}-i\,\cancel{X_2\cdot Y_1}+Y_1\cdot Y_2\right)\\ &=2(X_1\cdot X_2+Y_1\cdot Y_2)=2\,\mathrm{Re}\{Z_1\cdot Z_2^*\}\end{aligned}$$

i.e. $\mathrm{Re}\{\tilde{Z}_1\cdot\tilde{Z}_2^*\}=\mathrm{Re}\{\tilde{Z}_2\cdot\tilde{Z}_1^*\}$. We will explicitly prove this statement – the 1st term is:

$$\tilde{Z}_1\cdot\tilde{Z}_2^*=X_1\cdot X_2+iX_2\cdot Y_1-iX_1\cdot Y_2+Y\cdot_1 Y_2=\underbrace{(X_1\cdot X_2+Y_1\cdot Y_2)}_{=\mathrm{Re}\{\tilde{Z}_1\cdot\tilde{Z}_2^*\}}+i\underbrace{(X_2\cdot Y_1-X_1\cdot Y_2)}_{=\mathrm{Im}\{\tilde{Z}_1\cdot\tilde{Z}_2^*\}}$$

whereas the 2nd term (= changing in indices $1\rightleftarrows 2$ in the above expression) is:

$$\begin{aligned}\tilde{Z}_2\cdot\tilde{Z}_1^*&=X_2\cdot X_1+iX_1\cdot Y_2-iX_2\cdot Y_1+Y_2\cdot Y_1=X_1\cdot X_2+iX_1\cdot Y_2-iX_2\cdot Y_1+Y_1\cdot Y_2\\ &=\underbrace{(X_1\cdot X_2+Y_1\cdot Y_2)}_{=\mathrm{Re}\{\tilde{Z}_2\cdot\tilde{Z}_1^*\}}+i\underbrace{(X_1\cdot Y_2-X_2\cdot Y_1)}_{=\mathrm{Im}\{\tilde{Z}_2\cdot\tilde{Z}_1^*\}}\end{aligned}$$

Separately comparing the real and imaginary parts of each of these two terms, we see that indeed

$$\mathrm{Re}\{\tilde{Z}_1\cdot\tilde{Z}_2^*\}=\ \ \mathrm{Re}\{\tilde{Z}_2\cdot\tilde{Z}_1^*\}=(X_1\cdot X_2+Y_1\cdot Y_2)$$

Whereas: $$\mathrm{Im}\{\tilde{Z}_1\cdot\tilde{Z}_2^*\}=-\mathrm{Im}\{\tilde{Z}_2\cdot\tilde{Z}_1^*\}=(X_2\cdot Y_1-X_1\cdot Y_2).$$

Alternatively, we can equivalently see this another way, simply by working with the explicit expressions for complex $\tilde{Z}_1(\vec{r},t)=A_1(\vec{r},t)e^{i(\omega_1(t)t+\varphi_1(\vec{r},t))}$ and $\tilde{Z}_2(\vec{r},t)=A_2(\vec{r},t)e^{i(\omega_2(t)t+\varphi_2(\vec{r},t))}$:

$$\begin{aligned}\tilde{Z}_1\cdot\tilde{Z}_2^*+\tilde{Z}_2\cdot\tilde{Z}_1^*&=A_1e^{i(\omega_1t+\varphi_1)}\cdot A_2e^{-i(\omega_2t+\varphi_2)}+A_2e^{i(\omega_2t+\varphi_2)}\cdot A_1e^{-i(\omega_1t+\varphi_1)}\\ &=A_1\cdot A_2e^{i(\omega_1t+\varphi_1)}e^{-i(\omega_2t+\varphi_2)}+A_1\cdot A_2e^{i(\omega_2t+\varphi_2)}e^{-i(\omega_1t+\varphi_1)}\end{aligned}$$

Let us define $x\equiv(\omega_1(t)t+\varphi_1(t))$ and $y\equiv(\omega_2(t)t+\varphi_2(t))$. Rewriting the above expression:

$$\begin{aligned}\tilde{Z}_1\cdot\tilde{Z}_2^*+\tilde{Z}_2\cdot\tilde{Z}_1^*&=A_1\cdot A_2e^{ix}\cdot e^{-iy}+A_1\cdot A_2e^{iy}\cdot e^{-ix}\\ &=A_1\cdot A_2e^{i(x-y)}\quad +A_1\cdot A_2e^{i(y-x)}=A_1\cdot A_2\left(e^{i(x-y)}+e^{i(y-x)}\right)\\ &=A_1\cdot A_2\underbrace{\left(e^{i(x-y)}+e^{-i(x-y)}\right)}_{=2\cos(x-y)}=2\underbrace{A_1\cdot A_2\cos(x-y)}_{=\mathrm{Re}\{\tilde{Z}_1(t)\cdot\tilde{Z}_2^*(t)\}!!!}\\ &=2\,\mathrm{Re}\{\tilde{Z}_1\cdot\tilde{Z}_2^*\}\end{aligned}$$