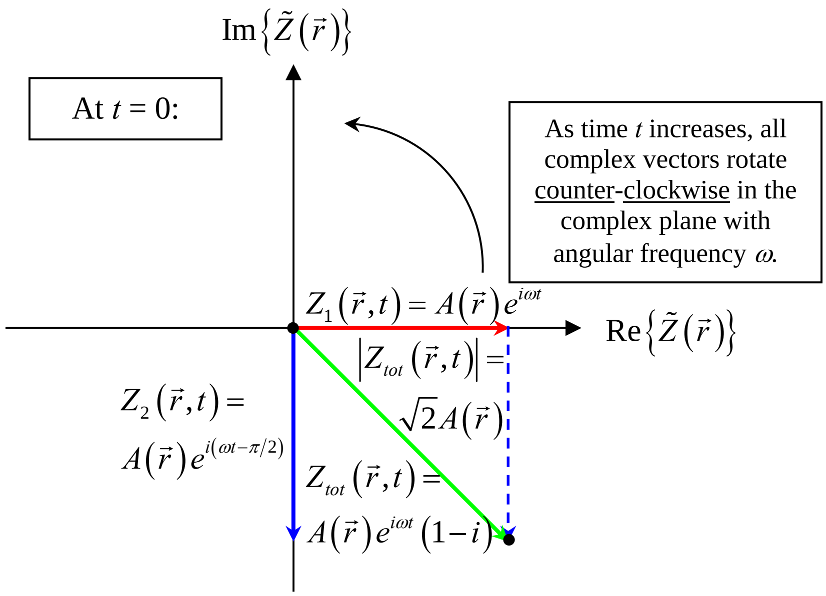

Note that the reason all complex vectors rotate in a counter-clockwise direction in the complex plane is due to the sign-choice of the $e^{+i\omega t} = \cos(\omega t) + i\sin(\omega t)$ time dependence – it determines the direction complex vectors rotate in the complex plane. Had we instead chosen the $e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t)$ time dependence, then all complex vectors would have instead rotated in a clockwise direction in the complex plane.

Throughout this course, note that we will ***always*** assume/adopt the convention of ***positive*** $e^{+i\omega t} = \cos(\omega t) + i\sin(\omega t)$ time dependence – because it turns out that the {default} way we use the lock-in amplifiers in the various phase-sensitive experiments that we have in the P406POM lab implicitly corresponds mathematically to the $e^{+i\omega t}$ convention – hence it is extremely important to use the correct mathematical descriptions in order to match experimental realities!

Note also that if we had instead chosen the second amplitude to be $\tilde{Z}_2(\vec{r},t) = A(\vec{r},t)e^{i(\omega t+\pi/2)}$, then the 2nd signal $\tilde{Z}_2(\vec{r},t)$ would *lead* (*i.e.* be ahead of) the 1st signal $\tilde{Z}_1(\vec{r},t) = Ae^{i\omega t}$ by 90° degrees in phase. For this situation, the total complex amplitude is:

$$\tilde{Z}_{tot}(\vec{r},t) = \tilde{Z}_1(\vec{r},t) + \tilde{Z}_2(\vec{r},t) = A(\vec{r},t)e^{i\omega t} + A(\vec{r},t)e^{i(\omega t+\pi/2)} = A(\vec{r},t)e^{i\omega t}\left(1+e^{+i\pi/2}\right)$$
$$= A(\vec{r},t)e^{i\omega t}\left[1+\left(\cancel{\cos(\pi/2)} + i\sin(\pi/2)\right)\right] = A(\vec{r},t)e^{i\omega t}(1+i)$$

with the same magnitude as before:

$$\left|\tilde{Z}_{tot}(\vec{r},t)\right| = \sqrt{\tilde{Z}_{tot}(\vec{r},t)\tilde{Z}^*_{tot}(\vec{r},t)} = A(\vec{r},t)\sqrt{(1+i)(1-i)} = A(\vec{r},t)\sqrt{1+\cancel{i}-\cancel{i}+1} = \sqrt{2}A(\vec{r},t)$$

We can now also see that a change in the sign of a complex quantity: $\tilde{Z}(t) \Rightarrow -\tilde{Z}(t)$ physically corresponds to a phase change/shift in phase/phase *retardation* of –180°
(*n.b.* which is also mathematically equivalent to a phase *advance* of +180°).
In other words: $\tilde{Z}'(\vec{r},t) = -\tilde{Z}(\vec{r},t) = \tilde{Z}(\vec{r},t)e^{\pm i\pi}$ because $e^{\pm i\pi} = \cos(\pi) \pm \cancel{i\sin(\pi)} = \cos(\pi) = -1$.