Linear Superposition (Addition) of Two Periodic Signals

It is illustrative to consider the situation associated with the linear superposition of two complex periodic, equal-amplitude, identical-frequency amplitudes at a given observation point $\vec{r}$ in 3-D space {defined from a local origin $O(0,0,0)$}, where one signal differs in relative phase from the other by $\varphi = -90^{\mathrm{o}} = -\pi/2$. Since the zero of time is arbitrary, we have the freedom to chose one signal to be purely real at time $t = 0$, *e.g.* such that:

$$\tilde{Z}_1(\vec{r},t) = A(\vec{r},t)e^{i\omega t} = A(\vec{r},t)\left(\cos(\omega t) + i\sin(\omega t)\right) \text{ with } \left|\tilde{Z}_1(\vec{r},t)\right| = A(\vec{r},t)$$

and the other signal,

$$\tilde{Z}_2(\vec{r},t) = A(\vec{r},t)e^{i(\omega t-\pi/2)} = A(\vec{r},t)\left(\cos(\omega t-\pi/2) + i\sin(\omega t-\pi/2)\right) \text{ with } \left|\tilde{Z}_2(\vec{r},t)\right| = A(\vec{r},t),$$

i.e. both signals have purely real amplitude, $A(\vec{r},t)$ and angular frequency, ω.

Note also, that at this point in the discussion, the two complex amplitudes $\tilde{Z}_1(\vec{r},t)$ and $\tilde{Z}_2(\vec{r},t)$ are (for the moment) taken to be "generic" acoustic quantities – *i.e.* both could represent *e.g.* complex pressure $\tilde{p}(\vec{r},t)$, complex particle velocity $\tilde{\vec{u}}(\vec{r},t)$, complex displacement $\tilde{\vec{\xi}}(\vec{r},t)$ and/or complex acceleration $\tilde{\vec{a}}(\vec{r},t)$.

At time $t = 0$:

$$\tilde{Z}_1(\vec{r},t=0) = A(\vec{r},t=0)e^{i0} = A(\vec{r},t=0)e^{0} = A(\vec{r},t=0)\left(\cos(0) + \cancel{i\sin(0)}\right) = A(\vec{r},t=0)$$

and:

$$\tilde{Z}_2(\vec{r},t=0) = A(\vec{r},t=0)e^{i(0+\pi/2)} = A(\vec{r},t=0)\left(\cancel{\cos(-\pi/2)} + i\sin(-\pi/2)\right) = -iA(\vec{r},t=0)$$

Thus, for this specific example, we see that the 2nd signal $\tilde{Z}_2(\vec{r},t) = A(\vec{r},t)e^{i(\omega t-\pi/2)}$ *lags* (*i.e.* is behind) the 1st signal $\tilde{Z}_1(\vec{r},t) = A(\vec{r},t)e^{i\omega t}$ by 90^{o} in phase, as shown in the figure below, for $t = 0$. The resultant/total complex amplitude $\tilde{Z}_{tot}(\vec{r},t)$ is the {instantaneous} *phasor* sum of the two individual complex amplitudes:

$$\tilde{Z}_{tot}(\vec{r},t) = \tilde{Z}_1(\vec{r},t) + \tilde{Z}_2(\vec{r},t) = A(\vec{r},t)e^{i\omega t} + A(\vec{r},t)e^{i(\omega t-\pi/2)} = A(\vec{r},t)e^{i\omega t}\left(1+e^{-i\pi/2}\right)$$

$$= A(\vec{r},t)e^{i\omega t}\left[1+\left(\cancel{\cos(-\pi/2)} + i\sin(-\pi/2)\right)\right] = A(\vec{r},t)e^{i\omega t}(1-i)$$

with magnitude:

$$\left|\tilde{Z}_{tot}(\vec{r},t)\right| = \sqrt{\tilde{Z}_{tot}(\vec{r},t)\tilde{Z}^*_{tot}(\vec{r},t)} = A(\vec{r},t)\sqrt{(1-i)(1+i)} = A(\vec{r},t)\sqrt{1-\cancel{i}+\cancel{i}+1} = \sqrt{2}A(\vec{r},t)$$