

The above $t = t$ phasor diagram has been rotated CCW by an angle $\varphi_o = \omega t$ relative to the $t = 0$ phasor diagram. We can thus rewrite the above matrix equation as:

$$\begin{pmatrix} \operatorname{Re}\{\tilde{R}_{out}(t)\} \\ \operatorname{Im}\{\tilde{R}_{out}(t)\} \end{pmatrix} = \begin{pmatrix} \cos \varphi_o & -\sin \varphi_o \\ \sin \varphi_o & \cos \varphi_o \end{pmatrix} \begin{pmatrix} R_{out}^o(\omega) \cos \varphi(\omega) \\ R_{out}^o(\omega) \sin \varphi(\omega) \end{pmatrix}$$

The 2×2 matrix $\begin{pmatrix} \cos \varphi_o & -\sin \varphi_o \\ \sin \varphi_o & \cos \varphi_o \end{pmatrix}$ is in fact none other than the 2-D **rotation matrix**, which takes a 2-D vector $\begin{pmatrix} X \\ Y \end{pmatrix}$ and rotates it {in a CCW direction} by an angle $\varphi_o = \omega t$ in the X - Y plane to $\begin{pmatrix} X' \\ Y' \end{pmatrix}$:

$$\begin{pmatrix} X' \\ Y' \end{pmatrix} = \begin{pmatrix} \cos \varphi_o & -\sin \varphi_o \\ \sin \varphi_o & \cos \varphi_o \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

precisely as shown in the above $t = t$ phasor diagram!

\Rightarrow See/hear complex sound demo using loudspeaker, p/u mics + 4 'scopes and 2 lock-in amplifiers...