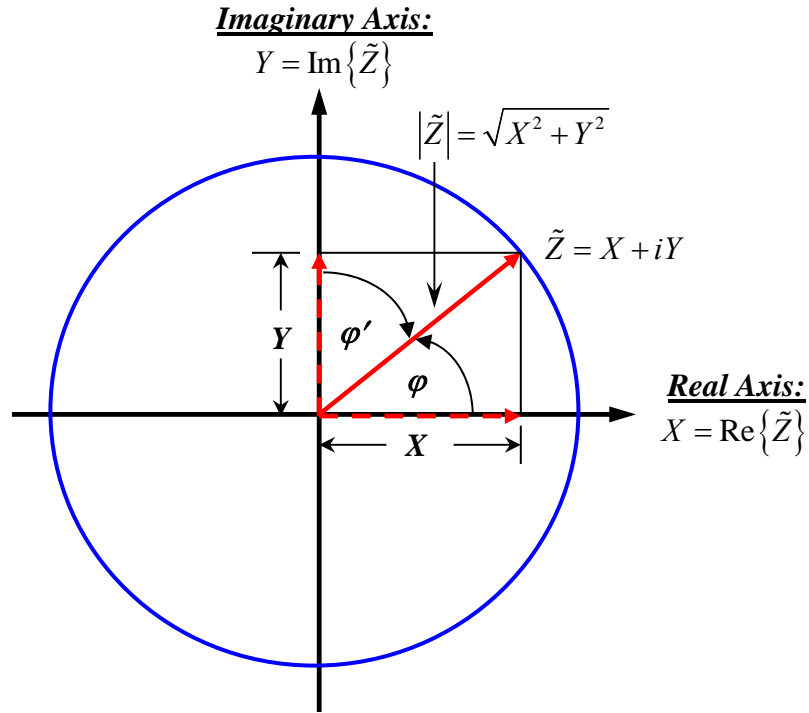


The *in-phase*, so-called “*real*” component of  $\tilde{Z}$ ,  $X = \text{Re}\{\tilde{Z}\}$  by convention is drawn along the  $x$ , or horizontal axis (*i.e.* the *abscissa*), as shown in the figure below.

The  $90^\circ$  *out-of-phase/quadrature*, so-called “*imaginary*” component of  $\tilde{Z}$ ,  $Y = \text{Im}\{\tilde{Z}\}$  by convention is drawn along the  $y$ , or vertical axis (*i.e.* the *ordinate*), as shown in the figure below.



It can be readily seen from the above diagram that the endpoint of the complex “vector” (*aka* “phasor”),  $\tilde{Z} = X + iY$  lies at a point on the circumference of a circle, centered at  $(X, Y) = (0, 0)$ , with “radius” (*i.e.* magnitude)  $|\tilde{Z}| = \sqrt{X^2 + Y^2}$  and phase angle,  $\varphi = \tan^{-1}(Y/X)$  (*n.b.* defined relative to the  $X$ -axis), (or equivalently:  $\varphi' = \tan^{-1}(X/Y)$ , *n.b.* defined relative to the  $Y$ -axis).

Instead of using Cartesian coordinates, we can alternatively/equivalently express the complex variable,  $\tilde{Z}$  in polar coordinate form:  $\tilde{Z} = |\tilde{Z}|(\cos \varphi + i \sin \varphi)$ , since from the above diagram, we see that  $X = |\tilde{Z}|\cos \varphi$  and  $Y = |\tilde{Z}|\sin \varphi$ . Recall the trigonometric identity:  $\cos^2 \varphi + \sin^2 \varphi = 1$  which is used in obtaining the magnitude of  $\tilde{Z}$ ,  $|\tilde{Z}|$  from  $\tilde{Z}$  itself:

$$\begin{aligned} |\tilde{Z}| &= \sqrt{\tilde{Z}\tilde{Z}^*} = |\tilde{Z}|\sqrt{(\cos \varphi + i \sin \varphi)(\cos \varphi - i \sin \varphi)} \\ &= |\tilde{Z}|\sqrt{\cos^2 \varphi + \cancel{i \sin \varphi \cos \varphi} - \cancel{i \sin \varphi \cos \varphi} + \sin^2 \varphi} \\ &= |\tilde{Z}|\sqrt{\cos^2 \varphi + \sin^2 \varphi} = |\tilde{Z}| \end{aligned}$$