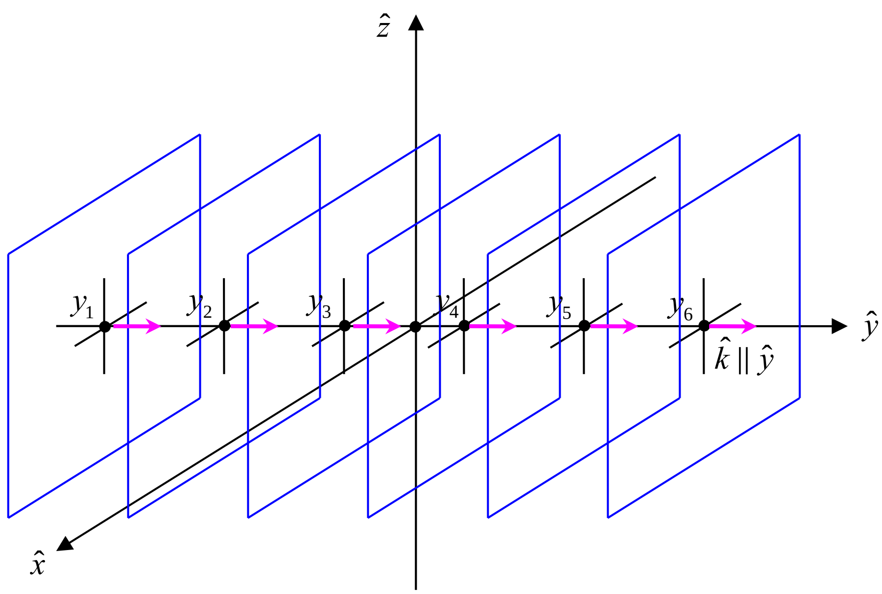
Surfaces of constant phase are $\vec{k}\cdot\vec{r} = k_x x + k_y y = constant$ or: $y = -\left(k_x/k_y\right)x + constant$, which is the equation of a straight line $y(x) = mx + b$ with slope: $m = -\left(k_x/k_y\right) = -\left(k\cos\theta/k\sin\theta\right) = -\cot\theta$ and y-intercept: $b = constant$.

At *e.g.* fixed $y = 0$, this traveling wave is: $\tilde{\psi}(\vec{r},t) = \tilde{\psi}(x,0,z,t) = Ae^{i(\omega t - k_x x)} = Ae^{i(\omega t - kx\cos\theta)}$.

At *e.g.* fixed $x = 0$, this traveling wave is: $\tilde{\psi}(\vec{r},t) = \tilde{\psi}(0,y,z,t) = Ae^{i(\omega t - k_y y)} = Ae^{i(\omega t - ky\sin\theta)}$.

The 3-D complex monochromatic traveling plane wave solution(s) to the above linear, homogeneous, 2[nd]-order differential equations also physically means that ***propagating*** 2-D planes (*aka* *wavefronts*) of constant phase $\varphi(\vec{r},t) = \omega t \mp \vec{k}\cdot\vec{r}$ also exist, as shown in the figure below, *e.g.* for a ***scalar*** 3-D complex monochromatic traveling plane wave propagating in the $\hat{k} = +\hat{y}$ direction with $\vec{k} = k_y\hat{y}$ and observer position $\vec{r} = y\hat{y}$, thus, here: $\tilde{\psi}(\vec{r},t) = Ae^{i(\omega t - \vec{k}\cdot\vec{r})} = Ae^{i(\omega t - k_y y)}$:



For each of the $i = 1:6$ planes located at $y = y_i$ in the above figure, at a specific instant in time, t the phase $\varphi_i(\vec{r},t) = \varphi(x, y = y_i, z, t) = \omega t - ky_i$ associated with the complex traveling plane wave propagating in the $\hat{k} = +\hat{y}$ direction is the same (*i.e.* constant) for ***every*** (x,z) point on that $y = y_i$ plane. Note also that the phase difference $\Delta\varphi_{i,i-1}(\vec{r},t)$ between successive planes i and i–1 is *also* constant, as well as time-independent:

$$\Delta\varphi_{i,i-1}(\vec{r},t) \equiv \varphi(x, y = y_i, z, t) - \varphi(x, y = y_{i-1}, z, t) = (\cancel{\omega t} - ky_i) - (\cancel{\omega t} - ky_{i-1}) = -k\underbrace{(y_i - y_{i-1})}_{\equiv\Delta y} = -k\Delta y$$