The complex monochromatic scalar and vector traveling plane waves $\tilde{\psi}(\vec{r},t)$, $\tilde{\vec{\psi}}(\vec{r},t)$ {obviously} must respectively satisfy the 3-D wave equations:

$$\nabla^2\tilde{\psi}(\vec{r},t)-\frac{1}{v_\varphi^2}\frac{\partial^2}{\partial t^2}\tilde{\psi}(\vec{r},t)=0 \text{ and: } \nabla^2\tilde{\vec{\psi}}(\vec{r},t)-\frac{1}{v_\varphi^2}\frac{\partial^2\tilde{\vec{\psi}}(\vec{r},t)}{\partial t^2}=0$$

In 3-D Cartesian/rectangular coordinates the ***Laplacian*** operator $\nabla^2 \equiv \vec{\nabla}\bullet\vec{\nabla}$ {where $\vec{\nabla}$ is the ***gradient*** operator} has the form:

$$\nabla^2 \equiv \vec{\nabla}\bullet\vec{\nabla} = \left(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right)\bullet\left(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right)=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

Explicitly carrying out the differentiation(s), we obtain the ***dispersion*** ***relation*** associated with propagation of a complex monochromatic traveling plane wave {in "***free air***"}:
$k_x^2+k_y^2+k_z^2=\omega^2/v_\varphi^2$, which since $k^2 \equiv |\vec{k}|^2=\vec{k}\bullet\vec{k}=k_x^2+k_y^2+k_z^2$ can be equivalently written as:
$k^2=\omega^2/v_\varphi^2$ or: $k=\omega/v_\varphi$, hence the phase velocity: $v_\varphi=\omega/k=f\lambda=constant \neq fcn(f,\vec{r},...)$.

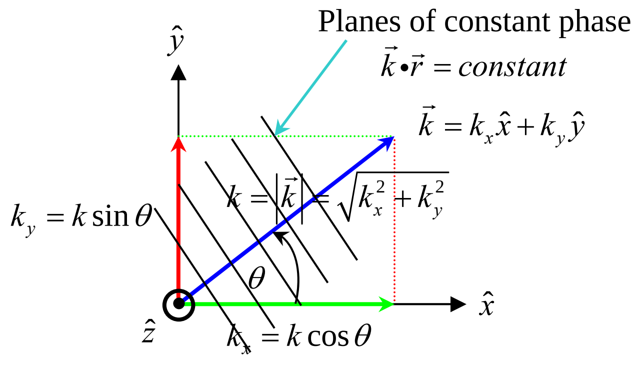
The surface{s} of ***constant phase*** associated with a traveling plane wave occur for $\vec{k}\bullet\vec{r}=constant$ in the argument of the $e^{i(\omega t\mp\vec{k}\bullet\vec{r})}$ factor in $\tilde{\psi}(\vec{r},t)$ and/or $\tilde{\vec{\psi}}(\vec{r},t)$.

From the fundamental/mathematical definition of a {spatial} gradient, the vector wavenumber:

$$\vec{k}\equiv\vec{\nabla}(\vec{k}\bullet\vec{r})=\left(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right)(k_x x+k_y y+k_z z)=k_x\hat{x}+k_y\hat{y}+k_z\hat{z}$$

is a vector quantity that points in a direction *perpendicular* to the surface(s) of constant phase, $\vec{k}\bullet\vec{r}=constant$. Physically, it points in the direction of propagation of the traveling plane wave.

If *e.g.* the vector wavenumber $\vec{k}$ lies only in the *x*-*y* plane {thus making an angle θ with respect to the $\hat{x}$-axis}, then $\tilde{\psi}(\vec{r},t)=Ae^{i(\omega t-\vec{k}\bullet\vec{r})}=Ae^{i(\omega t-k_x x-k_y y)}$ and 2-D planar surfaces of constant phase are oriented parallel to the $\hat{z}$-axis as shown in the figure below {for a "snapshot-in-time", *e.g.* at *t* = 0}:



From the above figure, we see that: $\vec{k}=k_x\hat{x}+k_y\hat{y}+\cancel{k_z\hat{z}}=k_x\hat{x}+k_y\hat{y}=k\cos\theta\,\hat{x}+k\sin\theta\,\hat{y}$.