Vector fields $\vec{\tilde{\psi}}(\vec{r},t)$ at each/every space-time point $(\vec{r},t)$ have ***do*** have an explicit direction associated with them – namely the $\hat{\tilde{\psi}}(\vec{r},t) = \vec{\tilde{\psi}}(\vec{r},t)\big/\left|\vec{\tilde{\psi}}(\vec{r},t)\right|$ direction. Thus, for a 3-D complex ***vector*** monochromatic traveling plane wave propagating in an arbitrary direction $\hat{k}$ in 3-D space:

$$\text{Prop. in } \hat{k}\text{-direction:} \quad \vec{\tilde{\psi}}(\vec{r},t) = \vec{A}\left\{\cos\left(\omega t \mp \vec{k}\bullet\vec{r}\right) + i\sin\left(\omega t \mp \vec{k}\bullet\vec{r}\right)\right\} = \vec{A}e^{i\left(\omega t \mp \vec{k}\bullet\vec{r}\right)}$$

where the complex vector ***amplitude*** associated with the 3-D complex vector monochromatic traveling plane wave $\vec{\tilde{\psi}}(\vec{r},t)$ propagating in an arbitrary direction $\hat{k}$ in 3-D space is given by:

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

The above expressions for 3-D complex ***scalar*** and ***vector*** monochromatic traveling plane waves propagating in an arbitrary direction $\hat{k}$ in 3-D space are formal mathematical solutions to their corresponding 3-D wave equations:

$$\nabla^2\tilde{\psi}(\vec{r},t) - \frac{1}{v_\varphi^2}\frac{\partial^2\tilde{\psi}(\vec{r},t)}{\partial t^2} = 0 \quad \text{and} \quad \nabla^2\vec{\tilde{\psi}}(\vec{r},t) - \frac{1}{v_\varphi^2}\frac{\partial^2\vec{\tilde{\psi}}(\vec{r},t)}{\partial t^2} = 0 \quad \text{respectively.}$$

The 3-D complex monochromatic traveling plane wave solution(s) to these two linear, homogeneous, 2nd-order differential equations physically correspond, respectively to ***scalar*** and ***vector*** waves propagating in the $\hat{k}$ direction in a 3-D medium which has the following physical properties:

(a) The medium is *lossless*, *i.e.* no friction/no damping and/or dissipative processes exist.

(b) The medium is also *dispersionless*, *i.e.* there is no frequency dependence of the phase speed of propagation v in the medium, *i.e.* $v_\varphi = constant \neq fcn(f,\vec{r},...)$, such that the ***dispersion relationship*** $v_\varphi = f\lambda = \omega/k = constant \neq fcn(f,\vec{r},...)$ is valid/holds in the medium.

If the medium ***is*** dissipative and/or dispersive, the above wave equation(s) and their solutions will necessarily be modified/change as a consequence of such phenomena.

Note also that the 3-D vector wave equation $\nabla^2\vec{\tilde{\psi}}(\vec{r},t) - \frac{1}{v_\varphi^2}\frac{\partial^2\vec{\tilde{\psi}}(\vec{r},t)}{\partial t^2} = 0$ is actually three *separate/independent* wave equations, since $\vec{\tilde{\psi}}(\vec{r},t) = \tilde{\psi}_x(\vec{r},t)\hat{x} + \tilde{\psi}_y(\vec{r},t)\hat{y} + \tilde{\psi}_z(\vec{r},t)\hat{z}$:

$$\nabla^2\tilde{\psi}_x(\vec{r},t) - \frac{1}{v_\varphi^2}\frac{\partial^2\tilde{\psi}_x(\vec{r},t)}{\partial t^2} = 0,\ \nabla^2\tilde{\psi}_y(\vec{r},t) - \frac{1}{v_\varphi^2}\frac{\partial^2\tilde{\psi}_y(\vec{r},t)}{\partial t^2} = 0 \text{ and } \nabla^2\tilde{\psi}_z(\vec{r},t) - \frac{1}{v_\varphi^2}\frac{\partial^2\tilde{\psi}_z(\vec{r},t)}{\partial t^2} = 0$$

i.e. $\nabla^2\vec{\tilde{\psi}}(\vec{r},t) - \frac{1}{v_\varphi^2}\frac{\partial^2\vec{\tilde{\psi}}(\vec{r},t)}{\partial t^2} = 0$

$$= \left[\nabla^2\tilde{\psi}_x(\vec{r},t) - \frac{1}{v_\varphi^2}\frac{\partial^2\tilde{\psi}_x(\vec{r},t)}{\partial t^2}\right]\hat{x} + \left[\nabla^2\tilde{\psi}_y(\vec{r},t) - \frac{1}{v_\varphi^2}\frac{\partial^2\tilde{\psi}_y(\vec{r},t)}{\partial t^2}\right]\hat{y} + \left[\nabla^2\tilde{\psi}_z(\vec{r},t) - \frac{1}{v_\varphi^2}\frac{\partial^2\tilde{\psi}_z(\vec{r},t)}{\partial t^2}\right]\hat{z} = 0$$