

In terms of the usual 3-D spherical-polar coordinate system's **polar** and **azimuthal** angles  $\theta$  and  $\varphi$ , respectively it is straightforward to show that:

$$\begin{aligned}\cos \Theta_x &= \sin \theta \cos \varphi, \\ \cos \Theta_y &= \sin \theta \sin \varphi \quad \text{and} \\ \cos \Theta_z &= \cos \theta, \quad \text{i.e. that } \Theta_z = \theta.\end{aligned}$$

Likewise, the wavevector  $\vec{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$  has its own direction cosines:

$$\begin{aligned}\cos \Theta_{k_x} &\equiv \hat{k} \cdot \hat{x} = k_x / \sqrt{k_x^2 + k_y^2 + k_z^2} = k_x / |\vec{k}| = k_x / k, \\ \cos \Theta_{k_y} &\equiv \hat{k} \cdot \hat{y} = k_y / \sqrt{k_x^2 + k_y^2 + k_z^2} = k_y / |\vec{k}| = k_y / k \quad \text{and} \\ \cos \Theta_{k_z} &\equiv \hat{k} \cdot \hat{z} = k_z / \sqrt{k_x^2 + k_y^2 + k_z^2} = k_z / |\vec{k}| = k_z / k.\end{aligned}$$

Note again that:  $|\hat{k} \cdot \hat{k}|^2 = \cos^2 \Theta_{k_x} + \cos^2 \Theta_{k_y} + \cos^2 \Theta_{k_z} = 1$ .

If we were to imagine 1-D complex monochromatic traveling plane waves propagating in the  $\hat{k}_x = \pm \hat{x}$ ,  $\hat{k}_y = \pm \hat{y}$  and  $\hat{k}_z = \pm \hat{z}$  -directions we would describe each of these mathematically as:

$$\text{Prop. in } \hat{k}_x = \pm \hat{x}\text{-direction: } \tilde{\psi}_x(\vec{r}, t) = A \left\{ \cos(\omega t \mp k_x x) + i \sin(\omega t \mp k_x x) \right\} = A e^{i(\omega t \mp k_x x)}$$

$$\text{Prop. in } \hat{k}_y = \pm \hat{y}\text{-direction: } \tilde{\psi}_y(\vec{r}, t) = A \left\{ \cos(\omega t \mp k_y y) + i \sin(\omega t \mp k_y y) \right\} = A e^{i(\omega t \mp k_y y)}$$

$$\text{Prop. in } \hat{k}_z = \pm \hat{z}\text{-direction: } \tilde{\psi}_z(\vec{r}, t) = A \left\{ \cos(\omega t \mp k_z z) + i \sin(\omega t \mp k_z z) \right\} = A e^{i(\omega t \mp k_z z)}$$

From these relations, noting that  $\vec{k} \cdot \vec{r} = (k_x \hat{x} + k_y \hat{y} + k_z \hat{z}) \cdot (x \hat{x} + y \hat{y} + z \hat{z}) = k_x x + k_y y + k_z z$ , we can generalize to the case for a complex monochromatic traveling plane wave propagating in an **arbitrary** direction  $\hat{k} = (k_x \hat{x} + k_y \hat{y} + k_z \hat{z}) / |\vec{k}| = (k_x \hat{x} + k_y \hat{y} + k_z \hat{z}) / k$  in 3-D space:

$$\text{Prop. in } \hat{k}\text{-direction: } \tilde{\psi}(\vec{r}, t) = A \left\{ \cos(\omega t \mp \vec{k} \cdot \vec{r}) + i \sin(\omega t \mp \vec{k} \cdot \vec{r}) \right\} = A e^{i(\omega t \mp \vec{k} \cdot \vec{r})}$$

The above expression for a complex monochromatic traveling plane wave propagating in an arbitrary direction  $\hat{k}$  in 3-D space is an appropriate description for a complex **scalar** field – e.g. complex pressure  $\tilde{p}(\vec{r}, t)$  – because **scalar** fields  $\tilde{\psi}(\vec{r}, t)$  at each/every space-time point  $(\vec{r}, t)$  have **no** explicit direction associated with them, other than their propagation direction  $\hat{k}$ .

We can also easily generalize the above complex scalar traveling wave description  $\tilde{\psi}(\vec{r}, t)$  to describe 3-D complex **vector** monochromatic traveling plane waves propagating in an arbitrary direction  $\hat{k}$  in 3-D space – e.g. complex particle displacement  $\vec{\xi}(\vec{r}, t)$ , particle velocity  $\vec{u}(\vec{r}, t)$  and/or particle acceleration  $\vec{a}(\vec{r}, t)$ . We can mathematically describe “generic” 3-D complex **vector** fields e.g. in Cartesian / rectangular coordinates in the following form:

$$\vec{\psi}(\vec{r}, t) = \tilde{\psi}_x(\vec{r}, t) \hat{x} + \tilde{\psi}_y(\vec{r}, t) \hat{y} + \tilde{\psi}_z(\vec{r}, t) \hat{z}.$$