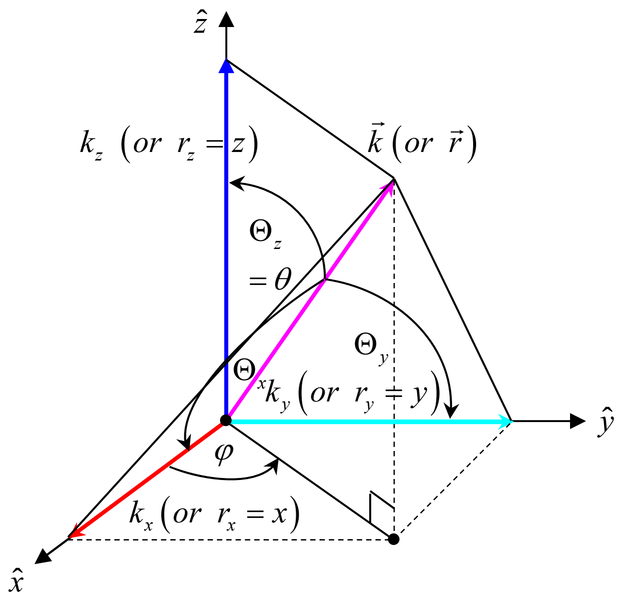
In order to describe monochromatic traveling plane waves propagating in an *arbitrary* direction in 3-D space, in analogy to the 3-D position vector $\vec{r} = r_x\hat{x} + r_y\hat{y} + r_z\hat{z} = x\hat{x} + y\hat{y} + z\hat{z}$, we introduce the concept of a ***wavevector*** $\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$. The wavevector $\vec{k}$ is an important physical quantity because it tells us the ***propagation direction*** of the wave – it is in the $\hat{k} = \vec{k}/|\vec{k}| = \vec{k}/k$ direction. The k_x, k_y, k_z are the components of the wavevector $\vec{k}$ along (*i.e.* ***projections*** onto) the $\hat{x}, \hat{y}, \hat{z}$ axes, respectively as shown in the figure below:



The ***magnitude*** of the observer's position vector $\vec{r}$ is: $r = |\vec{r}| = \sqrt{\vec{r}\cdot\vec{r}} = \sqrt{r_x^2 + r_y^2 + r_z^2} = \sqrt{x^2 + y^2 + z^2}$.

Likewise, the ***magnitude*** of the wavevector $\vec{k}$ is: $k = |\vec{k}| = \sqrt{\vec{k}\cdot\vec{k}} = \sqrt{k_x^2 + k_y^2 + k_z^2}$.

The three *x*,*y*,*z*-***direction cosines*** associated with the position vector $\vec{r}$ are obtained from dot products (*aka* ***inner products***) of the ***unit position vector*** $\hat{r}$ with the $\hat{x}, \hat{y}, \hat{z}$ axes, respectively:

$$\cos\Theta_x \equiv \hat{r}\cdot\hat{x}, \quad \cos\Theta_y \equiv \hat{r}\cdot\hat{y} \quad \text{and} \quad \cos\Theta_z \equiv \hat{r}\cdot\hat{z}$$

Since $\vec{r} = r\hat{r}$, then: $\hat{r} = \vec{r}/r = \vec{r}/|\vec{r}| = \vec{r}/\sqrt{x^2 + y^2 + z^2} = x\hat{x} + y\hat{y} + z\hat{z}/\sqrt{x^2 + y^2 + z^2}$.

Thus we see that:

$$\cos\Theta_x \equiv \hat{r}\cdot\hat{x} = x/\sqrt{x^2 + y^2 + z^2} = x/|r| = x/r,$$
$$\cos\Theta_y \equiv \hat{r}\cdot\hat{y} = y/\sqrt{x^2 + y^2 + z^2} = y/|r| = y/r \quad \text{and}$$
$$\cos\Theta_z \equiv \hat{r}\cdot\hat{z} = z/\sqrt{x^2 + y^2 + z^2} = z/|r| = z/r.$$

Note also that: $|\hat{r}\cdot\hat{r}|^2 = \cos^2\Theta_x + \cos^2\Theta_y + \cos^2\Theta_z = 1$.