If at time t = 0 the two phasors are <u>precisely</u> in phase with each other (*i.e.* with initial relative phase $\Delta \varphi_{21} = 0.0$), then the resultant/total amplitude, $\tilde{Z}_{tot}(\vec{r}, t = 0) = \tilde{Z}_1(\vec{r}, t = 0) + \tilde{Z}_2(\vec{r}, t = 0)$ at time t = 0 will be as shown in the figure below.

$$Z_{tot}(t=0) = Z_1(t=0) + Z_2(t=0)$$

$$Z_1(t=0) \qquad Z_2(t=0)$$

As time progresses, if $\omega_1 \neq \omega_2$, (noting that phasor 1 has angular frequency $\omega_1 = 2\pi f_1 = 2*1000\pi = 2000\pi$ radians/sec and phasor 2 has angular frequency $\omega_2 = 2\pi f_2 = 2*980\pi = 1960\pi$ radians/sec in our example above) phasor 1, with higher angular frequency will precess more rapidly than phasor 2 (by the <u>difference</u> in angular frequencies, $\Delta \omega = (\omega_1 - \omega_2) = (2000\pi - 1960\pi) = 40\pi$ radians/second). Thus as time increases, if $\omega_1 > \omega_2$, phasor 1 will <u>lead</u> phasor 2.

Eventually (at time $t = \frac{1}{2}\tau_{\text{beat}} = 0.025 = 1/40^{\text{th}}$ sec in our above example) phasor 2 will be lagging precisely $\Delta \varphi = \pi$ radians, or 180° behind in phase relative to phasor 1. At time $t = \frac{1}{2}\tau_{\text{beat}}$ = 0.025 sec = $1/40^{\text{th}}$ sec phasor 1 will be oriented <u>exactly</u> as it was at time t = 0.0 (having precessed exactly $N_1 = \omega_1 t / 2\pi = 2\pi f_1 t / 2\pi = f_1 t = 25.0$ revolutions in this time period), whereas phasor 2 will be pointing in the <u>opposite</u> direction at this instant in time (having precessed only $N_2 = \omega_2 t / 2\pi = 2\pi f_2 t / 2\pi = f_2 t = 24.5$ revolutions in this same time period), and thus the total amplitude $\tilde{Z}_{tot} (\vec{r}, t = \frac{1}{2}\tau_{beat}) = \tilde{Z}_1 (\vec{r}, t = \frac{1}{2}\tau_{beat}) + \tilde{Z}_2 (\vec{r}, t = \frac{1}{2}\tau_{beat})$ will be zero at this instant in time (if the magnitudes of the two individual amplitudes are precisely equal to each other), or minimal (if the magnitudes of the two individual amplitudes are <u>not</u> precisely equal to each other), as shown in the figure below.

$$Z_{\text{tot}}(t = \frac{1}{2}\tau_{\text{beat}}) = Z_1(t = \frac{1}{2}\tau_{\text{beat}}) + Z_2(t = \frac{1}{2}\tau_{\text{beat}}) = 0$$

$$Z_2(t = \frac{1}{2}\tau_{\text{beat}}) = -Z_1(t = \frac{1}{2}\tau_{\text{beat}})$$

As time progresses further, phasor 2 will continue to lag further and further behind phasor 1, and eventually (at time $t = \tau_{\text{beat}} = 0.050 \text{ sec} = 1/20^{\text{th}}$ sec in our above example) phasor 2, having precessed through $N_2 = 49.0$ revolutions will now be exactly $\Delta \varphi = 2\pi$ radians, or 360° (or one full revolution) behind in phase relative to phasor 1 (which has precessed through $N_1 = 50.0$ full revolutions), thus, the net/overall result here is the same as being exactly in phase with phasor 1! At this instant in time, $Z_{\text{tot}}(t = \tau_{\text{beat}}) = Z_1(t = \tau_{\text{beat}}) + Z_2(t = \tau_{\text{beat}}) = 2Z_1(t = \tau_{\text{beat}})$, and the phasor diagram at time $t = \tau_{\text{beat}}$ looks precisely like that as shown above for time t = 0.

Thus, it should (hopefully) now be clear to the reader that the phenomenon of beats is manifestly that of time-dependent alternating constructive/destructive interference between two periodic signals of comparable frequency, at the amplitude level. This is by no means a trivial point, as often the beats phenomenon is discussed in many physics textbooks in the context of intensity, $|\tilde{I}_{tot}(\vec{r},t)| \propto |\tilde{Z}_{tot}(\vec{r},t)|^2$. However, from the above discussion, it should be clear that the physics origin of the beats phenomenon has absolutely <u>nothing</u> to do with the <u>intensity</u> of the overall/resultant signal, it arises from wave interference at the <u>amplitude</u> level.

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