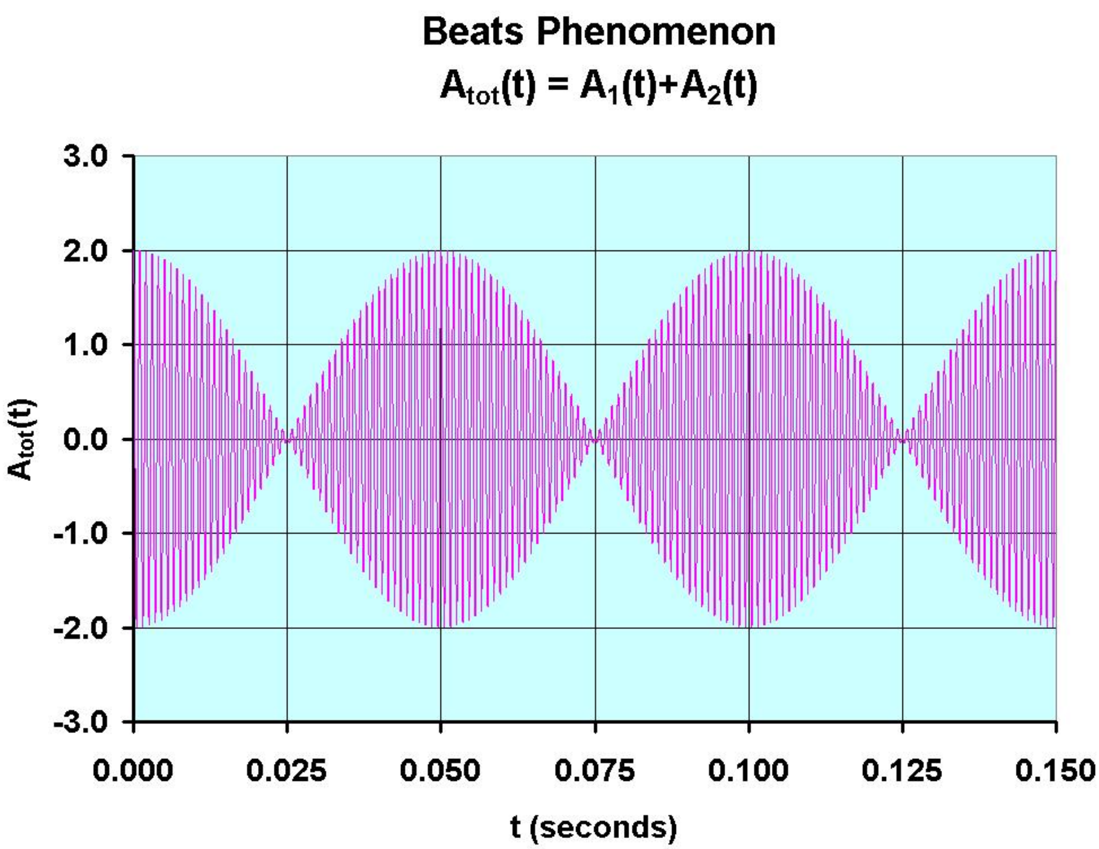
The magnitude of the total complex amplitude, $\left|\tilde{Z}_{tot}(\vec{r},t)\right| = \left|\tilde{Z}_1(\vec{r},t) + \tilde{Z}_2(\vec{r},t)\right|$ *vs*. time, *t* is shown in the figure below for time-independent/constant frequencies of f_1 = 1000 *Hz* and f_2 = 980 *Hz*, equal amplitudes of unit strength, *i.e.* $A_1(\vec{r},t) = A_2(\vec{r},t) = A(\vec{r},t) = A = 1.0$ and zero relative phase, $\Delta\varphi_{12}(\vec{r},t) = 0$:



The beats phenomenon can clearly be seen in the above waveform of the magnitude of the total amplitude $\left|\tilde{Z}_{tot}(\vec{r},t)\right| = \left|\tilde{Z}_1(\vec{r},t) + \tilde{Z}_2(\vec{r},t)\right|$ *vs*. time, *t*. From the above graph, it is obvious that the beat period, $\tau_{beat} = 1/f_{beat}$ = 0.050 sec = 1/20th sec, corresponding to a beat frequency, $f_{beat} = 1/\tau_{beat}$ = 20 *Hz*, which is simply the frequency difference, $f_{beat} \equiv |f_1 - f_2|$ between f_1 = 1000 *Hz* and f_2 = 980 *Hz*. Thus, the beat period, $\tau_{beat} = 1/f_{beat} = 1/|f_1 - f_2|$. When $f_1 = f_2$, the beat period becomes infinitely long, and thus no beats are heard!

In terms of the phasor diagram, as time progresses the individual amplitudes $\tilde{Z}_1(\vec{r},t)$ and $\tilde{Z}_2(\vec{r},t)$ precess (*i.e*. rotate) counter-clockwise in the complex plane at (angular) rates of $\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$ radians per second respectively, completing one revolution in the phasor diagram for each cycle/each period of $\tau_1 = 2\pi/\omega_1 = 1/f_1$ and $\tau_2 = 2\pi/\omega_2 = 1/f_2$, respectively.