

Room Resonances:

Resonances in a room/auditorium that occur at certain specific frequencies are simply the result of 1-D, 2-D or 3-D acoustic standing waves in the room/auditorium, analogous to the standing wave resonances that occur in 1-D systems such as closed-closed organ pipes.

The details of solving the 3-D wave equation

$$\nabla^2 \vec{\xi}(x, y, z, t) - \frac{1}{v^2} \frac{\partial^2 \vec{\xi}(x, y, z, t)}{\partial t^2} = 0$$

$$\nabla^2 p(x, y, z, t) - \frac{1}{v^2} \frac{\partial^2 p(x, y, z, t)}{\partial t^2} = 0$$

for either the 3-D vector particle displacement amplitude $\vec{\xi}(x, y, z, t)$ and/or scalar over-pressure amplitude $p(x, y, z, t)$ in a 3-D rectangular box (*i.e.* a room) of dimensions $\{L \times W \times H\} = \{L_x, L_y, L_z\}$ are discussed in detail on pages 3-7 of P406 POM lecture notes “Mathematical Musical Physics of the Wave Equation – Part 2”. The boundary conditions are that the displacement amplitude $\psi(x, y, z, t)$ has nodes on the surfaces {the walls/floor/ceiling} of the room, whereas the over-pressure amplitude $p_o(x, y, z, t)$ has anti-nodes on these surfaces.

Because three spatial dimensions are involved in this problem, three integer indices $\{l, m, n\}$ are required to uniquely describe the standing wave modes of the resonances in the room/auditorium. By convention, note that the $\{x, y, z\}$ indices $\{l, m, n\}$ are ordered strictly in decreasing room dimensions, *i.e.* by convention we have $\{L_x \geq L_y \geq L_z\}$.

There are 3 different basic types of modes of propagation of standing waves in a room.

- a.) 1-D or so-called **axial** modes, simply associated with the 1-D wave equation.
- b.) 2-D or so-called **transverse** modes, associated with the 2-D wave equation.
- c.) 3-D or so-called **oblique** modes, associated with the full 3-D wave equation.

For example, the lowest three 1-D **axial** modes, $[xyz] = [100]$, $[010]$ and $[001]$ or higher 1-D axial modes such as $[n00]$, $[0n0]$ or $[00n]$ where integer $n = 1, 2, 3, 4, 5, \dots$ are 1-D standing waves in either the x , y or z -directions that exist between two opposing parallel walls (or *e.g.* ceiling & floor) that are separated by a distance L_x , L_y or L_z , respectively with corresponding frequencies $f_{n00} = nv/2L_x$, $f_{0n0} = nv/2L_y$ or $f_{00n} = nv/2L_z$. The 3 orthogonal axial mode paths that can be taken by such standing waves are indicated with arrows as shown in the figure below: