<u>Speed</u>: = instantaneous time rate of change of position = <u>magnitude</u> of velocity:

$$v(\vec{r},t) = \left|\vec{v}(\vec{r},t)\right| = \sqrt{v_x^2(\vec{r},t) + v_y^2(\vec{r},t) + v_z^2(\vec{r},t)}$$ (Cartesian Coordinates)

Thus, <u>Velocity</u> = instantaneous speed in a given <u>direction</u>, *e.g.* in east direction, or up, or down, *etc*.

From calculus, we know that the instantaneous velocity $\vec{v}(\vec{r},t)$ is the partial derivative of the instantaneous position with respect to time (= instantaneous <u>*slope*</u> of $\vec{r}(t)$ *vs*. *t* graph):

Velocity: $$\vec{v}(\vec{r},t) = \frac{\partial \vec{r}(t)}{\partial t} = \frac{\partial x(t)}{\partial t}\hat{x} + \frac{\partial y(t)}{\partial t}\hat{y} + \frac{\partial z(t)}{\partial t}\hat{z} = v_x(\vec{r},t)\hat{x} + v_y(\vec{r},t)\hat{y} + v_z(\vec{r},t)\hat{z}$$

<u>Acceleration</u>: = instantaneous time rate of change of velocity, and a direction (up, down, east, west, *etc*.) specifying the direction in which the time rate of change of velocity is occurring. 3-D vector quantity (*SI* units = meters per second squared, *i.e.* m/s^2)

Speed increasing with time —— accelerating
Speed <u>decreasing</u> with time —— <u>decelerating</u>

Acceleration: $$\vec{a}(\vec{r},t) = a_x(\vec{r},t)\hat{x} + a_y(\vec{r},t)\hat{y} + a_z(\vec{r},t)\hat{z} = \partial\vec{v}(\vec{r},t)/\partial t$$ (Cartesian Coordinates)

Magnitude (size) of instantaneous acceleration: $$a(\vec{r},t) = \left|\vec{a}(\vec{r},t)\right| = \sqrt{a_x^2(\vec{r},t) + a_y^2(\vec{r},t) + a_z^2(\vec{r},t)}$$

From calculus, we also know that the instantaneous acceleration is the partial derivative of the instantaneous velocity with respect to time (= instantaneous <u>slope</u> of $\vec{v}(\vec{r},t)$ vs. *t* graph):

Acceleration: $$\vec{a}(\vec{r},t) = \frac{\partial \vec{v}(\vec{r},t)}{\partial t} = \frac{\partial v_x(\vec{r},t)}{\partial t}\hat{x} + \frac{\partial v_y(\vec{r},t)}{\partial t}\hat{y} + \frac{\partial v_z(\vec{r},t)}{\partial t}\hat{z} = a_x(\vec{r},t)\hat{x} + a_y(\vec{r},t)\hat{y} + a_z(\vec{r},t)\hat{z}$$

Motion in 3-D is independent in *x-y-z* directions for a <u>free particle</u> (unless geometrically <u>constrained</u> somehow – *e.g.* bead on a helix or circular ring):

3-D Equations of motion of a <u>free particle</u> with <u>constant acceleration</u>: $\vec{a}(\vec{r},t) = \vec{a}_o$

$$\vec{v}(\vec{r},t) = \vec{v}_o + \vec{a}_o t$$ (v_o = 3-D vector velocity at time $t = 0$)

$$\vec{r}(t) = \vec{r}_o + \vec{v}_o t + \tfrac{1}{2}\vec{a}_o t^2$$ (r_o = 3-D vector position at time $t = 0$)

Short-hand way to write out the separate *x-y-z* equations of motion (decouple for a free particle):

$$v_x(\vec{r},t) = v_{ox} + a_{ox}t \qquad x(t) = x_o + v_{ox}t + \tfrac{1}{2}a_{ox}t^2$$
$$v_y(\vec{r},t) = v_{oy} + a_{oy}t \qquad y(t) = y_o + v_{oy}t + \tfrac{1}{2}a_{oy}t^2$$
$$v_z(\vec{r},t) = v_{oz} + a_{oz}t \qquad z(t) = z_o + v_{oz}t + \tfrac{1}{2}a_{oz}t^2$$