rod is oriented at an angle, $\phi = \Omega t$, as shown in the above figure (note that we define the zero of time such that at $t = 0$, $\phi = 0$). Then we can write the Doppler shift formula for the frequency associated with each end of the rod as:

$$f_1' = \left(\frac{v}{v - v_s \cos\phi} \right) f = \left(\frac{v}{v - v_s \cos\Omega t} \right) f$$

for the end (# 1) of the rod moving directly towards the observer/listener at t = 0 (when $f_1' > f$) and:

$$f_2' = \left(\frac{v}{v + v_s \cos\phi} \right) f = \left(\frac{v}{v + v_s \cos\Omega t} \right) f$$

for the end (# 2) of the rod moving directly away from the observer/listener at t = 0 (when $f_2' < f$). Thus, the rotating vibrating rod actually emits two time-dependent frequencies, one slightly higher and one slightly lower than the original frequency, f for a stationary, non-rotating rod. Because of the small difference in frequency between f_1' and f and f_2' and f, the two Doppler-shifted frequencies, f_1' and f_2' are also similar to each other.

When two sounds are superimposed upon each other that differ from each other only slightly in frequency, the resultant, overall sound is one which is equivalent to a sound which has the average of the two frequencies, $\langle f_{avg} \rangle = ½ (f_1' + f_2')$ but which is *amplitude modulated* at the *difference* frequency, $\Delta f = | f_1' - f_2'|$. This acoustical phenomenon is known as beats, and is shown in the figures below, for two rotating sound sources as given by the above expressions for the two time-dependent frequencies, f_1' and f_2' for a rotating vibrating aluminum rod, of length L = 1.52 meters, fundamental frequency f = 1672 Hz, rotating at frequency of f_{rot} = 1 revolution/second, with rotational period $\tau_{rot} = 1/ f_{rot}$ = 1 second.

At a rotational frequency of f_{rot} = 1 revolution/second, the maximum (minimum) Doppler-shifted frequencies are ~ 23 Hz higher (lower) than the stationary, non-rotational fundamental (n = 1) frequency of f = 1672 Hz, respectively. Thus, the maximum difference frequency due to the rotational motion associated with the Doppler effect is $\Delta f = | f_1' - f_2'|$ ~ 46 Hz, when the vibrating rod is *perpendicular* to the observer/listener's line of sight. However, as the vibrating rod rotates, the Doppler-shifted frequencies f_1' and f_2' also change with time. When the vibrating rod is oriented such that it is *parallel* to the line of sight of the observer/listener (this occurs at two times – at
$t = ¼\, \tau$ = 0.25 sec and at $t = ¾\tau$ = 0.75 sec), at those moments in time when there is no Doppler shift of either sound source (from the listener's perspective) and hence no beats are heard at that instant in time, since the two frequencies are identical, the beat frequency between them $\Delta f = | f_1' - f_2'|$ = 0 Hz. When the rod is again oriented perpendicular to the observer/listener's line of sight (at $t = ½\, \tau$ = 0.5 sec and at $t = \tau$ = 1.0 sec), then the Doppler shifts high and low are again maximal, with the maximal beat frequency! The two frequencies, f_1'(t) and f_2'(t) as a function of time, t for one entire rotational period, are shown in the figure below.