

We summarize the modal frequencies, wavelengths, and the locations of nodes and anti-nodes for the first nine harmonics associated with a vibrating rod of length,  $L$  in the table below.

Harmonic Mode #, $n$	Frequency $f$ (Hz)	Wavelength $\lambda$ (m)	Node Locations, (m)	Anti-Node Locations, (m)
1	$f_1$	$\lambda_1 = 2L$	0	$\pm 1/2L$
2	$f_2 = 2f_1$	$\lambda_2 = 1/2\lambda_1 = L$	$\pm 1/4L$	$0, \pm 2/4L = \pm 1/2L$
3	$f_3 = 3f_1$	$\lambda_3 = 1/3\lambda_1 = 2/3L$	$0, \pm 2/6L = \pm 1/3L$	$\pm 1/6L, \pm 3/6L = \pm 1/2L$
4	$f_4 = 4f_1$	$\lambda_4 = 1/4\lambda_1 = 1/2L$	$\pm 1/8L, \pm 3/8L$	$0, \pm 2/8L = \pm 1/4L, \pm 4/8L = \pm 1/2L$
5	$f_5 = 5f_1$	$\lambda_5 = 1/5\lambda_1 = 2/5L$	$0, \pm 2/10L = \pm 1/5L, \pm 4/10L = \pm 2/5L$	$\pm 1/10L, \pm 3/10L, \pm 5/10L = \pm 1/2L$
6	$f_6 = 6f_1$	$\lambda_6 = 1/6\lambda_1 = 1/3L$	$\pm 1/12L, \pm 3/12L = \pm 1/4L, \pm 5/12L$	$0, \pm 2/12L = \pm 1/6L, \pm 4/12L = \pm 1/3L, \pm 6/12L = \pm 1/2L$
7	$f_7 = 7f_1$	$\lambda_7 = 1/7\lambda_1 = 2/7L$	$0, \pm 2/14L = \pm 1/7L, \pm 4/14L = \pm 2/7L, \pm 6/14L = \pm 3/7L$	$\pm 1/14L, \pm 3/14L, \pm 5/14L, \pm 7/14L = \pm 1/2L$
8	$f_8 = 8f_1$	$\lambda_8 = 1/8\lambda_1 = 1/4L$	$\pm 1/16L, \pm 3/16L, \pm 5/16L, \pm 7/16L$	$0, \pm 2/16L = \pm 1/8L, \pm 4/16L = \pm 1/4L, \pm 6/16L = \pm 3/8L$
9	$f_9 = 9f_1$	$\lambda_9 = 1/9\lambda_1 = 2/9L$	$0, \pm 2/18L = \pm 1/9L, \pm 4/18L = \pm 2/9L, \pm 6/18L = \pm 1/3L, \pm 8/18L = \pm 4/9L$	$\pm 1/18L, \pm 3/18L, \pm 5/18L, \pm 7/18L, \pm 9/18L = \pm 1/2L$

We used a Hewlett-Packard HP-3652A Dynamic Signal Analyzer (another piece of electronic measurement equipment in our lab) to carry out real-time Fast-Fourier Transform/Harmonic Analysis, in order to measure the harmonic content of the modal vibrations of our  $L = 1.52$  m long aluminum rod. When the aluminum rod was held at its mid-point and excited, as expected, we observed the first three of the odd-harmonic ( $n = 1, 3, 5$ ) modes of vibration of the rod, which we measured to be at  $f_1 = 1672$  Hz,  $f_3 = 5012$  Hz ( $\sim 3 f_1$ ) and  $f_5 = 8350$  Hz ( $\sim 5 f_1$ ), respectively. Nearly all of the vibrational energy of the aluminum rod ( $> 99\%$ ) is in the fundamental – very little energy is contained in the higher ( $n = 3, n = 5$ ) harmonics. Most of this is in the 3<sup>rd</sup> harmonic, with even less in the 5<sup>th</sup> harmonic. Higher order ( $n = 7, 9, \dots$ ) harmonics were not observable with our setup.

When the rod was held at  $x = 1/4 L$  in order to excite (only) the 2<sup>nd</sup> harmonic, we observed (only) the second ( $n = 2$ ) harmonic at  $f_2 = 3338$  Hz ( $\sim 2 f_1$ ).

When the rod was held at  $x = 1/3 L$ , in order to excite (only) the 3<sup>rd</sup> harmonic odd harmonic, we observed (only) the third ( $n = 3$ ) harmonic at  $f_3 = 5012$  Hz ( $\sim 3 f_1$ ).