The next higher, fifth harmonic mode of vibration of the rod (n = 5) is shown in the figure below. The frequency  $f_5$  is five times higher than that of the fundamental frequency,  $f_1$ , since the wavelength,  $\lambda_4 = \frac{2}{4L} = \frac{1}{2}L$  for this mode of vibration of the rod is one fourth of that of the wavelength,  $\lambda_1 = 2L$  associated with the fundamental mode. This mode of vibration of the rod has five nodes, one at x = 0, two nodes located at  $x = \pm^{1}/_{5}L$ , and two others located at  $x = \pm^{2}/_{5}L$ . There are six anti-nodes, two located at  $x = \pm^{1}/_{10}L$ , two located at  $x = \pm^{3}/_{10}L$  and two located at the endpoints, at  $x = \pm^{5}/_{10}L = \pm^{1}/_{2}L$ .



There in fact exists an infinite hierarchy of so-called *normal modes of vibration* of the rod. Note that all modes (n = 0, 1, 2, 3, 4, 5, ...) of vibration of the rod all have the *same* longitudinal speed of propagation of sound in the rod,

$$v = f_1 \lambda_1 = f_2 \lambda_2 = f_3 \lambda_3 = \dots = f_n \lambda_n$$

the frequencies of the higher modes are integer multiples of the fundamental mode,  $f_n = n f_1$ , where n = 1, 2, 3, 4, 5, ... The wavelengths associated with the higher modes of vibration are related to the wavelength of the fundamental mode by  $\lambda_n = \lambda_1/n = 2L/n$ .

When a person excites the rod by holding the rod at its mid-point with one hand and pulling on it with rosin-dusted thumb and index fingers of the other hand, not only the fundamental is excited, but in fact the third, fifth, seventh, ninth, ... – all odd-*n* harmonics (n = 1, 3, 5, 7, 9, ...) are also excited. Note that the odd harmonics all have a node at the mid-point of the rod, x = 0, where it is held.

If the rod is held at  $x = \pm^{1/4} L$  to excite the 2<sup>nd</sup> harmonic, it can be seen that this location is at an anti-node of the 4<sup>th</sup> harmonic – thus the 4<sup>th</sup> harmonic cannot be simultaneously excited by holding the rod at this point. Only if harmonics simultaneously have a common node at a given location along the length of the rod, will it then be possible to simultaneously excite more than one such harmonic of the rod.