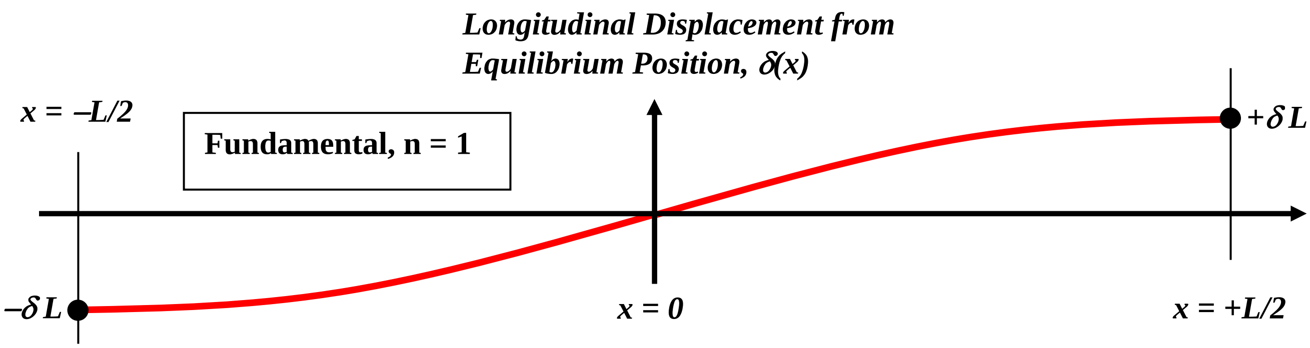
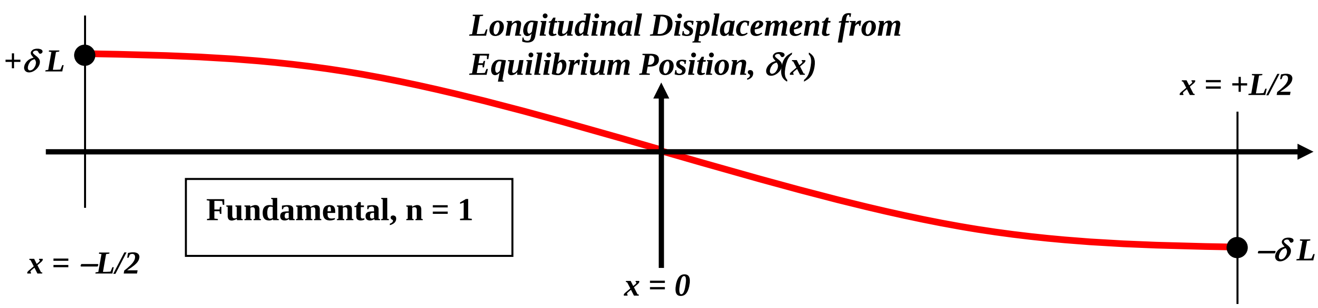


One half cycle later in the oscillation of the rod, the longitudinal displacement would appear as:



A further one half cycle later, the longitudinal displacement will again be as shown in the top picture above, and so on, as time progresses. At half-way times in between these two moments, the longitudinal displacement from equilibrium position is momentarily zero everywhere along the rod. The fundamental mode of vibration of the rod has one node, at its mid-point, $x = 0$.

The (longitudinal) speed of propagation of sound in the metal rod, v is given by the formula: where $Y = \sigma/\varepsilon$ = Young's modulus, also known as the *tensile elastic* modulus. It is the ratio of

$$v = \sqrt{\frac{Y}{\rho}}$$

longitudinal, compressive stress, $\sigma = F/A$ (longitudinal compressive force per unit cross sectional area of the rod) to the longitudinal compressive strain, $\varepsilon = |L_2 - L_1|/L_1$ where L_1 is the equilibrium length of the rod, and L_2 is the extended length of the rod when stretched. The density (mass per unit volume) of the rod, is denoted by ρ. Aluminum has a density of $\rho_{AL} = 2.71\ \text{gm/cm}^3 = 2710\ \text{kg/m}^3$ and has a Young's modulus of $Y_{AL} = 70\times10^9\ \text{N/m}^2$. Thus, the speed of sound in aluminum rod is thus $v_{AL} = 5082.4$ m/s.

For an aluminum rod measured to be $L = 1.52$ meters long, the fundamental mode of vibration corresponds to a wavelength of $\lambda_1 = 2L = 3.04$ meters. From the relationship between propagation speed, frequency and wavelength, namely that $v = f_1\lambda_1$, then for the fundamental mode of vibration of the aluminum rod, we thus have $f_1 = v/\lambda_1 = (5082.4\ \text{m/s})/(3.04\ \text{m}) = 1671.8$ Hz (cycles per second).