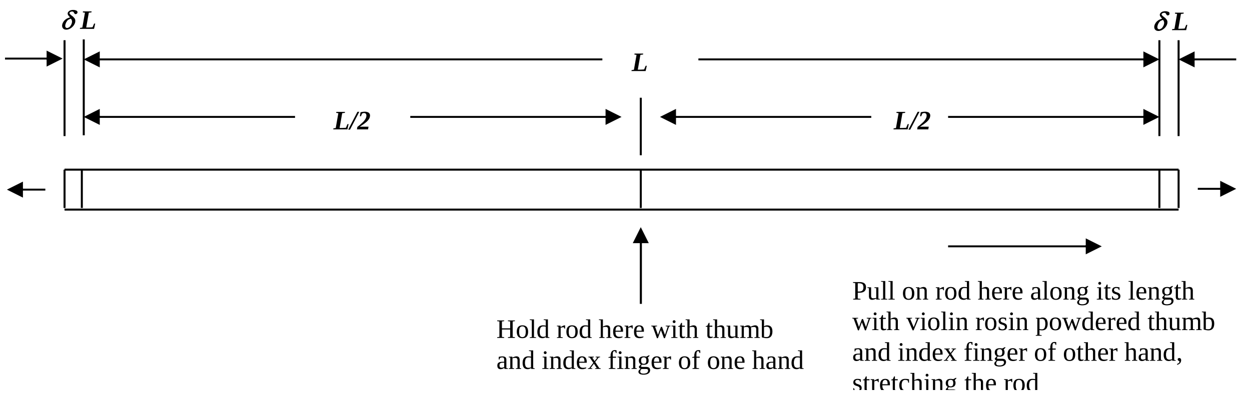
The Physics of a Longitudinally Vibrating "Singing" Metal Rod:

A metal rod (*e.g*. aluminum rod) a few feet in length can be made to vibrate along its length – make it "sing" at a characteristic, resonance frequency by holding it precisely at its mid-point with thumb and index finger of one hand, and then pulling the rod along its length, toward one of its ends with the thumb and index finger of the other hand, which have been dusted with crushed violin rosin, so as to obtain a good grip on the rod as it is pulled.



The pulling motion of the thumb and index finger actually stretches the rod slightly, giving it *potential energy* – analogous to the potential energy associated with stretching a spring along its length, or a rubber band. The metal rod is actually an *elastic* solid – elongating slightly when pulled! Pulling on the rod in this manner *excites* the rod, causing *both* of its ends to simultaneously vibrate longitudinally, back and forth along its length at a characteristic resonance frequency known as its fundamental frequency, f_1. For an excited aluminum rod of length, $L \sim 2$ meters, it is thus possible that at one instant in time both ends of the rod will be extended a small distance, $\delta L \sim 1$ mm beyond the normal, (i.e. non-stretched) equilibrium position of the ends of the rod. At another instant, one-half cycle later, the ends or the rod are compressed inwards this same amount. The displacement amplitude is a maximum at the ends of the rod. A point along the vibrating rod where the displacement amplitude is a maximum is known as a displacement *anti-node*. Thus both ends of the rod are displacement anti-nodes.

This type of excitation of a metal rod is known as the so-called fundamental, or first harmonic ($n = 1$) mode of excitation, or vibration – because this mode of vibration has the *lowest* possible frequency of vibration. The rod of length L vibrates in its fundamental mode with one-half of a wavelength, i.e. $L = ½\,\lambda_1$. The longitudinal displacement from equilibrium, along the length of the rod, as a function of position, is shown in the figure below. It can be seen that at the mid-point of the rod, the displacement amplitude is zero for this mode of vibration of the rod. A point along the vibrating rod where the displacement amplitude is zero is known as a ***node***. This is why the rod is held ***precisely*** at its mid-point. If the rod is held ***near*** to, but not ***at*** its mid-point, this mode of excitation of the rod is much harder to accomplish, and it is also quite rapidly damped out – the vibrational energy that is present in the rod is absorbed in one's hand where it is held. Precisely at the mid-point of the rod, there is no net displacement at that point, hence there is no way energy can be transferred from the rod to one's hand; thus the rod "sings" for a very long time, gradually decaying away from energy loss associated with direct radiation of sound waves into the air, and internal frictional dissipation processes associated with the finite stiffness of the rod.